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GENERAL INSTABILITY OF CIRCUMFERENTIALLY STIFFENED SANDWICH SHELLS SUBJECTED TO UNIFORM EXTERNAL PRESSURE

1961

JAROSLAW DRAHOMIR STACHIW

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The Pennsylvania State University

The Graduate School

Department of Engineering Mechanics

General Instability of Circumferentially Stiffened Sandwich Shells Subjected to Uniform External Pressure

A thesis in

Engineering Mechanics

by

Jaroslaw Drahomir Stachiw

Submitted in partial fulfillment
of the requirements
for the degree of

Master of Science

June 1961

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LIST OF SYMBOLS

SYMBOLS

R	Radius of neutral axis of ring
R_o	Radius of external surface of ring
R_i	Radius of internal surface of ring
t_o	External flange thickness of ring
t_i	Internal flange thickness of ring
t_w	Web thickness of ring
h	Depth of ring
I_{nx}	Moment of inertia of ring about neutral axis
A_x	Cross section of ring
p	External hydrostatic pressure, psi
p_{ce}	External hydrostatic collapse pressure of the shell assembly, experimentally determined
p_{ct}	External hydrostatic collapse pressure of the shell assembly, theoretically determined
q_{ct}	External radial collapse pressure of the ring, theoretically determined
p_{cts}	External hydrostatic collapse pressure of an "I" ring, theoretically determined
p_{ctr}	External hydrostatic collapse pressure of a shell and adaptor ring theoretically determined
p_{ctj}	External hydrostatic collapse pressure of a shell joint, theoretically determined
L_r	Width of ring flange

SYMBOLS

L	Length of shell between bulkheads
E	Modulus of elasticity in compression
E_t	Tangent modulus of elasticity in compression
μ	Poisson's ratio, at zero strain
μ_s	Poisson's ratio, at a given strain level
u	Radial displacement during buckling
D	Mean diameter of the shell
σ_{uc}	Ultimate strength in compression
σ_{yp}	Yielding stress in simple compression, 0.02 per cent offset
V_i	Displacement volume of shell, per unit shell length
W_i	Weight of shell, per unit length
f_s	Static coefficient of friction
f_r	Radial friction restraint opposing the deflection of shell end
t	Thickness of a smooth shell wall

ABSTRACT

A simple existing equation for the buckling of rings under radial pressure is adapted in this thesis for the calculation of external hydrostatic pressure at which circumferentially stiffened sandwich shells collapse by the mechanism of general instability.

Several circumferentially stiffened sandwich shells were designed, built and tested to destruction, and the external hydrostatic pressures causing general instability were compared with the shell collapse pressures predicted on the basis of an extension of Bresse's theory developed by the author. That is,

$$p_{ct} = \frac{3E_t I_{nx}}{(1-\mu_s^2)(R^2)(R_o)} \qquad (8)$$

Equation (8) gives the pressure at which an infinitely long circumferentially stiffened sandwich shell collapses due to general instability. This equation is recommended for use only when the h/D ratio of a circumferentially stiffened shell is less than 0.1.

CHAPTER I

INTRODUCTION

1.1 GENERAL STATEMENT OF THE PROBLEM

For many engineering applications it is necessary to have pressure vessels subjected to external pressure with higher pressure to weight ratio than that given by smooth ring stiffened cylindrical shells. This thesis, recognizing the superior pressure to weight ratio of sandwich shells, concerns itself with the resistance of these shells to collapse under external hydrostatic pressure.

Circumferentially stiffened cylindrical sandwich shells may fail by various mechanisms of collapse when subjected to uniform external hydrostatic pressure. Generally speaking, there are two broad categories of shell failure: (1) failure by elastic instability, and (2) failure by yielding or fracture. The shell's collapse due to elastic instability is the less predictable of the two above mentioned categories, and thus, of greater practical interest.

One of the many types of elastic instability possible is general instability, a type of elastic failure in which all of the shell components fail simultaneously by buckling. It is

desirable for safety reasons to be able to predict the magnitude of external pressure at which general elastic instability will occur.

The purpose of this investigation is to determine, both experimentally and theoretically, what relationship exists between the physico-mechanical shell parameters and the external hydrostatic pressure that causes the shell to collapse in a general instability mode.

1.2 ORIGIN AND IMPORTANCE OF THE STUDY

Pressure vessels subjected to external pressure, until the late nineteen hundreds, have received little attention in engineering circles. It was felt that the smooth cylindrical tube provided a cheap and yet very satisfactory type of structure for vessels that were subjected to external pressure. The construction materials, mostly cast iron or steel, were cheap and of inferior quality. For this reason relatively thick walled tubes were always used in external pressure vessels. The thick walls and low pressures used never permitted the stressing of the tubes to their elastic instability and thus, presented no stimulus for research and experimentation in this area.

It was only in 1880 that Bresse[95] derived an expression for buckling of rings and in 1888 Bryan[96] developed an accurate

expression for the elastic buckling of long tubes subjected to uniform external pressure. F. Engesser[98] extended the use of this formula both to materials that do not follow Hooke's law, and to plastic region of materials that do follow Hooke's law. Until the outbreak of World War I, this simple formula satisfied the needs of the engineering profession.

During World War I, there appeared in large quantities one of the deadliest weapons invented by man - the submarine. Its ability to lurk undetected underwater while delivering deadly blows to surface shipping made it one of the decisive weapons in a global conflict. Larger and deeper going submarines became the order of the day. As long as the submarines were limited to shallow dives, no structural problems arose, but when at the end of the war, they were expected to submerge to 250 feet depths, hull collapse due to elastic buckling became very prevalent.

The reason for these failures was lack of understanding of the complicated relationship between frame stiffness, plating thickness and frame spacing. The stock remedy, that of increasing the design safety factor, could not be applied here since the submarine needed every pound of positive buoyancy that it could achieve. Spurred on by this crisis in submarine design, Von Mises[108] developed a theory for the buckling of

shells between frames. Von Sanden, Kiel and Gunther[111] then developed a formula for the calculation of stresses in the shell at the frames and midway between frames. The size of the frames and the collapse of the shell due to general elastic instability was not determined then and it took another World War to answer these questions.

In the lull between the two World Wars, research and experimentation continued to test previously postulated theories and to fill in the remaining voids in the knowledge about ring stiffened shells. Trilling[119], Windenburg[121], Donnell[97], and Tokugawa[118] performed many experiments in which the shells were subjected to bending, compressive, and implosive loading. The previously postulated theories by Von Mises[109] and Von Sanden[111] were found to predict the experimental results well, although not completely satisfactorily. The theory of Von Mises[109], in particular, was found to predict accurately the number of lobes formed during buckling of tubes but, according to Batdorf[93], to differ significantly from experimental collapse pressure in the low curvature parameter region (for $\frac{L^2}{R_o t}\sqrt{1-\mu^2} < 100$ the theoretical values are as much as 50 per cent higher then experimental values).

It was during World War II and its immediate aftermath that the remaining gaps in the knowledge about shell collapse

were filled in. The general elastic instability of ring stiffened shells was solved by Kendrick[104, 105, 106] and the distribution of stresses in the shell and stiffeners by Salerno and Pulos[110]. With the contribution of Lunchick's[107] theory on plastic failure of ring stiffened shells, the failures of that type of shell, both in the elastic and plastic stress regions, became well understood. Now that the mechanisms of collapse of the ring stiffened shells were well understood, the shells could be designed to withstand a given pressure with a minimum of weight by the optimization of shell parameters. But even with all of the shell parameters optimized, the pressure to weight ratio of ring stiffened shells left much to be desired, and the idea arose that by using some other type of shell design, the pressure to weight ratio could possibly be significantly improved.

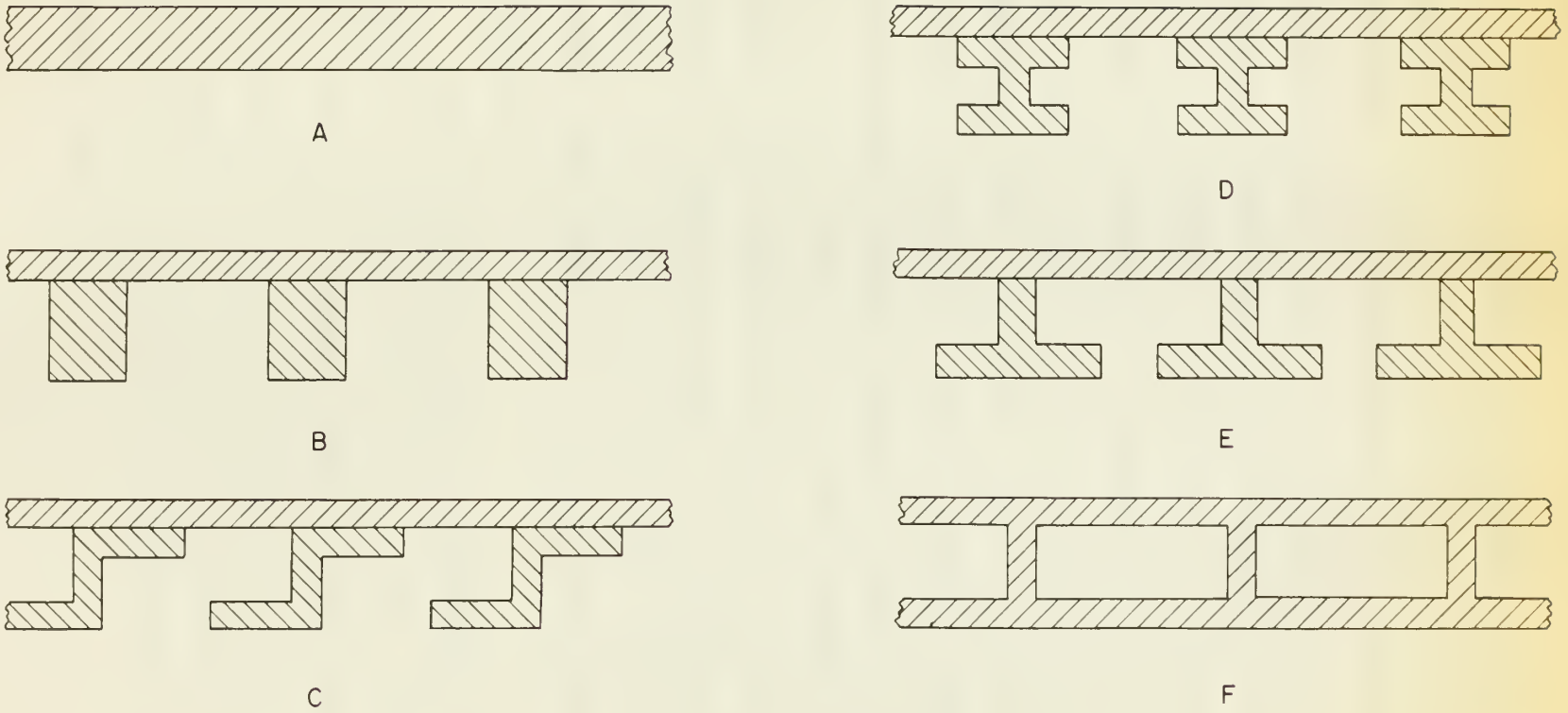
As the understanding of ring stiffened cylindrical shells increased, it became more and more apparent to interested observers, both here and abroad, that the ability of the shell to withstand external pressure was dependent basically only on two physical parameters, (1) the strength of the construction material, and (2) the rigidity of the stiffened wall. There was no shortage of commercial high strength materials. If only the proper shell structure was found in

which these high strength materials could be utilized to their maximum advantage, a shell of superior pressure to weight ratio would result. The talents of many engineers produced a number of ideas, some of which were rather bizarre. After the chaff and flotsam were drawn, one new idea for shell design appeared that promised to improve significantly the pressure to weight ratio of shells under external pressure. The new concept in shell design is the sandwich wall shell that due to its high rigidity makes the use of high strength construction materials profitable (Fig. 1). Although the idea of sandwich shells appeared on the engineering scene only in the late nineteen forties, it has already entrenched itself firmly in the minds of shell designers as the optimum shell design for external pressure.

1.3 PREVIOUS RELATED STUDIES

The idea of using sandwich walled shells grew out of the previous work with sandwich panels. Although sandwich panels were proposed a long time ago, they never saw wide usage until some of the sandwich fabrication techniques were perfected which made the sandwich panel competitive with other methods of construction. Brazing and bonding opened new vistas for sandwich panels and, later on, for sandwich shells.

STIFFENERES FOR VESSELS
SUBJECTED TO EXTERNAL PRESSURE



ALL WALL CROSS-SECTIONS ABOVE
HAVE THE SAME SECTION AREA AND DEPTH
BUT DIFFER IN RIGIDITY.
SECTION F REPRESENTS THE MOST RIGID CONSTRUCTION.

FIGURE 1

As the fabrication methods became perfected and engineers across the country began to use the sandwich type structure for different applications, it became imperative to derive, both on a theoretical and an experimental basis, stress-strain and stress-load relationships for sandwich structures of different configurations. In the short span of time from 1940 to 1960, the theory for flat sandwich plates became stabilized and well supported by a host of experimental data. References 1 through 39 give a concise description of the most recent and important theoretical and experimental results pertaining to sandwich plates. For an extensive discussion of previous sandwich panel and sandwich shell studies one should consult Ref. 43, which served this thesis as the basis of the discussion of previous studies. For a quick review and complete bibliography on sandwich panels, Refs. 4, 13, 34, and 35 are also highly recommended.

When the sandwich shells are considered, however, a completely different view presents itself. There are only a few researchers working on sandwich shells and experimental data supporting the available theories are lacking. Most of the research in this field has been conducted at the Forest Products Laboratory, New York University and Soviet research institutions.

The earliest work in the sandwich shell field was done by Leggett[66] in Britain, Reissner[81] in the United States and Panov[75] in U. S. S. R. The major contribution was made by Reissner[81], who constructed a non-buckling theory of small deflections and strains of sandwich shells, taking into account deflections not only due to shear but also due to compression of the sandwich core normal to the facings.

Stein and Mayer[82], following studies by Reissner, came up with a linear small deflection theory that does not consider core compression but includes average shear strains. Their theory, in terms of shears and deflections normal to the median surface, is expressed in three general equations with ten independent physical constants. The three equations of Stein and Mayer[82] can be reduced to Donnell's equation[97, 93], if the simplification is introduced that the sandwich core is isotropic and does not carry the stresses directly. Other workers in the field applied Stein and Mayer's[82] equations to different sandwich shell configurations under different types of loadings[83, 45, 88, 46, 92].

The research group working at New York University produced some outstanding results. The members of this group such as Teichman, Gerard, Wang, DeSanto, Vaccaro, Gould and others conducted a large number of theoretical and experimental

studies pertaining to the sandwich shells. Their biggest contribution in the theoretical field is a theory for the symmetrical buckling of sandwich shells under compressive end loads. Gerard[44] and later Wang[87], using equilibrium and conservation of energy principles and including shear effects, derived a generalized non-linear buckling theory. The basic assumptions of this theory are that, while neglecting the rigidity of the facings, the core behaves as a three dimensional elastic medium in which magnitude of stresses, whose direction is parallel to the facings, are negligible when compared to the normal and transverse shear stresses.

The experiments performed by the New York University group had as their object the comparison of the modified Donnell's equation (as given by Stein and Mayer[82]) and their own theoretical solutions[44, 84, 87, 85, 86, 45, 88, 46, 92, 91, 47, 49, 90] with experimental data. Wang and DeSanto[90] give a very good summary of the analytical and experimental investigations performed by this group. It can be stated that the modified Donnell's equation as derived by Stein and Mayer[82] for cylinders with a weak core predicts the buckling axial loads well, while the correlation between bending and torsional loads and the experimental data is not as good. However, until better theories are postulated, it must be said

that the linear buckling theory is acceptable for sandwich cylinders with soft cores under bending and torsional loadings.

The Forest Products Laboratory, although primarily interested in the structural applications of wood fiber board and plywood, has consistently contributed over the years to both analytical and experimental work on sandwich panels and cylinders. To describe but a sampling of their work one has to consider reports by Kuenzi, March, Raville and Haft. Building on, and extending previous work of the Forest Laboratory, Raville[79, 80] concludes that when the sandwich facings are relatively thin, an analysis treating the facings as membranes is sufficiently accurate. However, he finds that if the facings become thick then the stiffness of the facings has to be considered. The analytical work by Raville and others[61, 63, 64, 68, 69, 70, 71, 72, 74, 78] centered primarily on cylinders under axial and lateral loads.

Soviet scientists, realizing the potential applications of sandwich structures, have spent a considerable amount of effort on it as can be judged by the excellent papers by Prusakov[26], Vlasov[37], Grigoliuk[50-60], Korolev[62], Kurshin[65], Ambartsumian[38] and others[75]. From these, the work of Grigoliuk has produced the most generalized set of equations for sandwich shells. His generalized equations also make provisions for some plastic effects[60].

Looking over the rather hurried and scanty review of the work done on sandwich cylinders by workers here and abroad, one realizes how little is actually known about the detailed stress distribution in the sandwich cylinders and the various mechanisms of plastic and elastic collapse. Even when generalized sets of equations, like those of Grigoliuk and Donnell are available, it is quite difficult for the engineer-designer to use them for solutions of actual engineering problems. The one exception is Ref. 43 which presents some of the equations in graphical form for steel shells. While there is unquestionably an acute need for generalized stress-strain and deflection equations for orthotropic unsymmetric sandwich cylinders, there is also a very pressing need for special equations applicable to specific engineering applications. This thesis is concerned with the solution of a particular problem, the failure by general instability of circumferentially stiffened sandwich shells.

1.4 SPECIFIC STATEMENT OF THE PROBLEM

The circumferentially stiffened sandwich shell, like any other type of shell, is subject to failure both in the elastic and plastic strain regions of the construction

material. Because this type of construction is new, proven design equations have not been developed that accurately predict the failure of circumferentially stiffened shells under hydrostatic external pressure.

This thesis has as its primary aim the analytical development and experimental proof of a formula that will approximately predict the collapse pressure of circumferentially stiffened sandwich shells under external hydrostatic pressure.

The general instability collapse of such sandwich shells constitutes only one of the many modes of failure possible, but it is felt that the analytical derivation and experimental data collected in the preparation of this thesis will help designers to prevent this type of sandwich shell failure.

1.5 HYPOTHESES TO BE INVESTIGATED

It is postulated that the modified Bresse[95] equation for buckling of circular rings (Eq. 8) predicts accurately (within 5 per cent) the uniform external pressure at which failure of a long circumferentially stiffened sandwich shell will occur, by collapse mechanism of general instability.

1.6 DEFINITIONS OF TERMS

Circumferentially Stiffened Cylindrical Sandwich Shell - Sandwich shell of cylindrical shape whose inner and outer facings are tied together into a homogeneous structure by a series of circumferential annular rings spaced an equal distance apart. This shell is sometimes referred to in the literature as a cellular shell.

Annular Rings - Rings tying the outside and inside shell facings into a homogeneous structure.

Shell Facings - Thin, membrane-like skin covering the skeleton of annular rings both on the outside and on the inside of the shell.

"I" Ring - Typical shell section ring that has the configuration of a wide flange "I" beam rolled into a circular ring shape.

"I" Ring Flange - That portion of the outside and inside shell facing that belongs to the typical shell "I" ring.

"I" Ring Web - Dimension of the annular ring that is obtained when the annular ring is measured along the shell's axial direction.

Pressure to Weight Ratio - Arbitrary ratio giving an index for the comparison of various pressure vessel shells subjected to external or internal pressure. This comparison index of

shells takes into account both the strength to weight ratio of the structural material and the buckling resistance of a given shell design. The ratio is defined as:[120]

$$\eta = P_{ce}\left(\frac{V_i}{W_i}\right)$$

Hydrostatic Pressure - External pressure of uniform magnitude applied both axially and radially to the enclosed pressure vessel made up of one or many shells joined together.

Collapsing Pressure - External pressure that causes the pressure vessel to lose its structural integrity.

Infinitely Long Pressure Vessel - Pressure vessel possessing such bulkhead spacing that any further increase in the distance between the bulkheads will not decrease the collapse pressure of the vessel.

Short Pressure Vessel - Pressure vessel whose collapse pressure depends to some extent on the reinforcing action of the bulkheads.

Failure by General Instability - Type of failure in which all of the shell's structural components fail simultaneously by buckling.

General Instability Equation - Bresse's[95] theory for the buckling of rings has been adapted by the author in Eq. (8) for the calculation of external hydrostatic pressure at which

an infinitely long circumferentially stiffened sandwich shell will collapse due to general instability.

1.7 SCOPE AND LIMITATION OF THE STUDY

The theoretical part of this thesis consists of the derivation of Bresse ring buckling formula and its modification for use on circumferentially stiffened sandwich shells. The experimental part of this study deals with the design and implosion testing of several shells as described above.

The theoretically predicted external collapse pressure is compared to the experimentally obtained collapse pressure. The criterion by which the validity of the modified Bresse equation is the ability of the Eq.(8) to predict experimental collapse pressure within engineering design tolerances, which, in this study, are defined to be $\pm$ 10 per cent of the experimental results.

The investigation is limited to the collapse by general instability only. Local buckling of sandwich shell facings or of circumferential stiffeners is only discussed in a general way. Only those shell parameters and experimental data are presented which have direct bearing on the collapse mechanism investigated.

CHAPTER II

PROCEDURE OF INVESTIGATION

2.1 EXPERIMENTAL SHELLS

For the comparison between theoretical collapse pressures and the actual collapse pressure values several carefully designed and built circumferentially stiffened sandwich shells are required. The shells are the specimens whose behavior under load would show whether the postulated general instability collapse theory actually predicts the collapse pressures as postulated. Since each circumferentially stiffened sandwich shell represents a large investment of money, considerable thought had to be devoted to their design so that by destroying only a few of them sufficient experimental data would be supplied to prove or disprove Eq. (8), the general instability equation.

2.1.1 Acrylic Resin Shell Comparison Series

Before any expensive circumferentially stiffened sandwich shell design program was initiated, it was considered proper to design and construct a series of inexpensive exploratory shells, incorporating the representative types of shell designs found in the literature.

This was done in order to substantiate the claim made by the author and others that the circumferentially stiffened sandwich shell is worth investigating since it represents a more rigid shell structure than other shells presently built.

The series of exploratory shells consisted of five acrylic resin shells of identical weight, length, outside diameter, inside usable diameter, and method of shell end support (Figs. 2-8). The shells were fabricated from commercially available tubes, and rings cut from commercial acrylic resin sheet stock. All the structural components were joined into a single homogeneous structure by bonding with acrylic resin solvent. To eliminate residual stresses introduced by machining and bonding of the material, the finished shell assemblies were annealed in properly temperature-controlled ovens.

For the implosion testing of shells, their ends were sealed off with friction type end closures and immersed in a small 1600 psi capacity pressure chamber. Each shell was subjected to external hydrostatic pressure until implosion of the shell occurred. The implosion pressures for all the shells were carefully recorded and form the basis of comparison between the rigidities of individual shells, as shown in Table I.

TABLE I

IMPLOSION PRESSURES OF INDIVIDUAL SHELLS

OF

THE ACRYLIC RESIN SHELL COMPARISON SERIES

Material Properties: See Fig. 33

Pressurization Rate: 20 psi/second

Type	Description	Collapse Pressure	η*
Model 1 -	Smooth Shell	590 psi.	$.585 \times 10^5$
Model 2 -	Smooth Shell Stiffened by Equally Spaced Circumferential Plain Rings	1200 psi.	1.185×10^5
Model 3 -	Smooth Shell Stiffened by Equally Spaced Circumferential "T" Rings	1100 psi.	1.09×10^5
Model 4 -	Longitudinally Stiffened Sandwich Shell	1100 psi.	1.09×10^5
Model 5 -	Circumferentially Stiffened Sandwich Shell	1650 psi.	1.63×10^5

* Where η = pressure to weight ratio = $\frac{p_{ce} V_i}{W_i}$

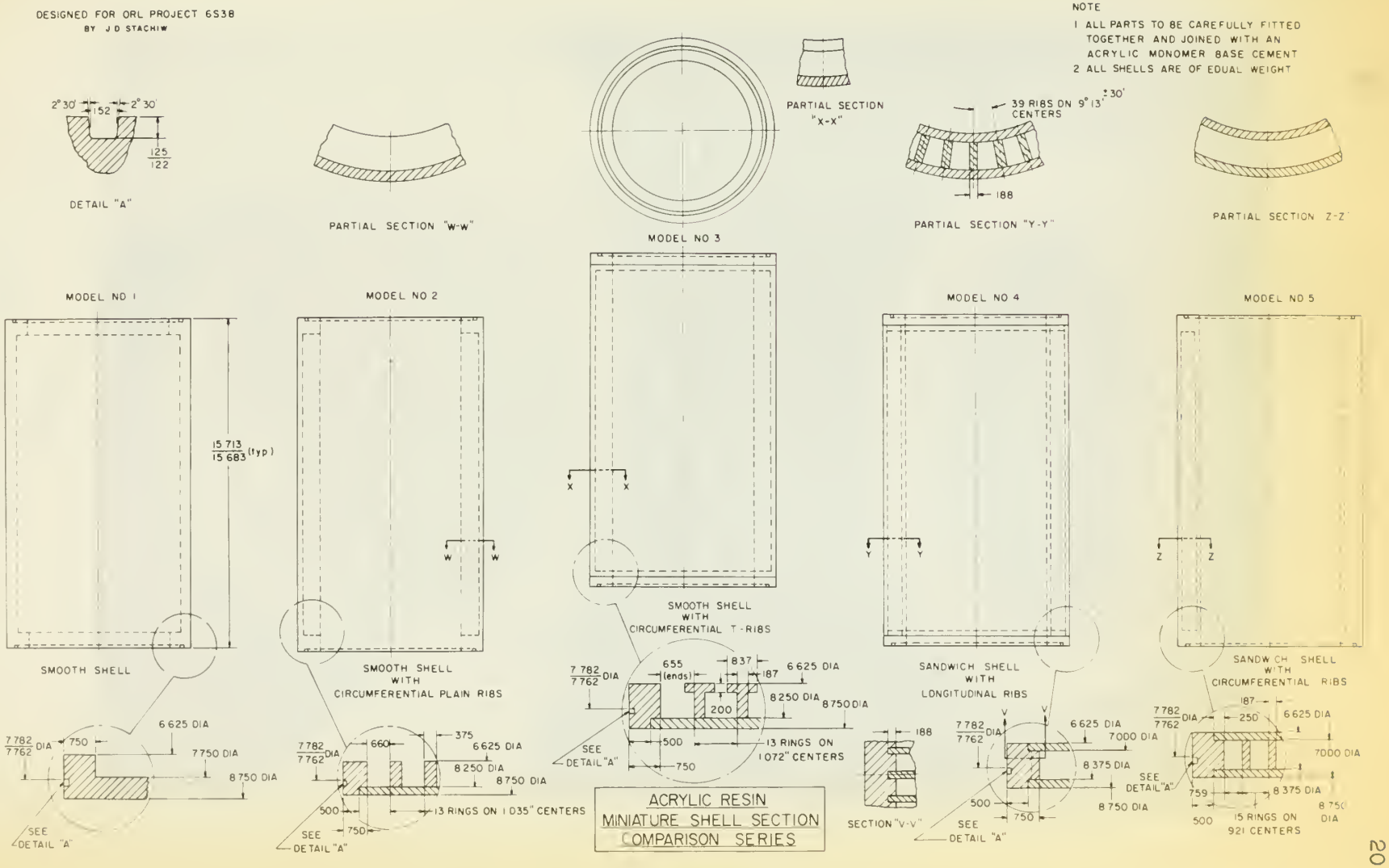

DESIGNED FOR ORL PROJECT 6538
BY J D STACHIW
NOTE
1 ALL PARTS TO BE CAREFULLY FITTED TOGETHER AND JOINED WITH AN ACRYLIC MONOMER BASE CEMENT
2 ALL SHELLS ARE OF EQUAL WEIGHT
2° 30'
2° 30'
152
125
122
DETAIL "A"
PARTIAL SECTION "W-W"
PARTIAL SECTION "X-X"
39 RIBS ON 9° 13' ± 30' CENTERS
188
PARTIAL SECTION "Y-Y"
PARTIAL SECTION Z-Z
MODEL NO 3
MODEL NO 1
MODEL NO 2
MODEL NO 4
MODEL NO 5
15 713
15 683 (typ)
SMOOTH SHELL
SMOOTH SHELL WITH CIRCUMFERENTIAL PLAIN RIBS
SMOOTH SHELL WITH CIRCUMFERENTIAL T-RIBS
SANDWICH SHELL WITH LONGITUDINAL RIBS
SANDWICH SHELL WITH CIRCUMFERENTIAL RIBS
7 782
7 762 DIA
750
6 625 DIA
7 750 DIA
8 750 DIA
SEE DETAIL "A"
660
375
8 250 DIA
500
750
13 RINGS ON 1 035" CENTERS
655 (ends)
837
187
200
8 750 DIA
13 RINGS ON 1 072" CENTERS
SECTION "V-V"
7 000 DIA
8 375 DIA
250
759
15 RINGS ON 921 CENTERS
ACRYLIC RESIN
MINIATURE SHELL SECTION
COMPARISON SERIES

FIGURE 2

FIGURE 3

MODEL 1, SMOOTH SHELL

FIGURE 5

MODEL 3, SMOOTH SHELL STIFFENED BY EQUALLY SPACED CIRCUMFERENTIAL "T" RIBS

FIGURE 6

MODEL 4, LONGITUDINALLY STIFFENED SANDWICH SHELL

FIGURE 7

MODEL 5, STRUCTURAL COMPONENTS OF THE CIRCUMFERENTIALLY STIFFENED SANDWICH SHELL

FIGURE 8

MODEL 5, READY FOR IMPLOSION TESTING

The experimental data from the series of acrylic resin shells confirms that the circumferentially stiffened sandwich shell is the best method of stiffening shells against external pressure. It then definitely became worth - while to investigate further the structural stability of this type of shell.

2.1.2. Aluminum Sandwich Shells

To test the general instability collapse theory postulated in this thesis, two larger circumferentially stiffened sandwich shells were designed on the basis of Eq. (8). The principal aim in the design of the two larger shells was to obtain a shell structure that would collapse only by the mechanism of general instability, the mode of failure considered in this thesis. In this way experimental confirmation is provided for the proposed general instability theory expressed in Eq. (8). This means that the thickness of the shell facings and spacing of annular rings had to be selected on such a basis that no local buckling or yielding would occur before the shell collapsed as a whole under the action of external pressure. Since engineering design formulas are lacking for these shells, both the shell facings and the rib spacing were selected on the basis of general engineering stability principles[113, 114].

The circumferentially stiffened sandwich shells* were made of wrought metal so that experimental verification could be provided using a typical engineering type construction material. The particular alloy was selected solely on the basis of cost and ease of fabrication.

Since so many variables enter into the construction of a sandwich shell, it is not prudent to accept the experimental collapse pressure of a single shell as the typical collapse pressure of that sandwich shell design. The best approach to obtaining the typical collapse pressure would be to test as many shells as possible of the same design, and then to evaluate the collapse pressures by statistical methods. Although sound statistically, the approach would be too expensive for this investigation. To get at least a semblance of a typical collapse pressure, it was decided to make both aluminum shells of identical dimensions so that by testing them and averaging their collapse pressures, an average value could be obtained. The dimensional tolerances for the fabrication of both

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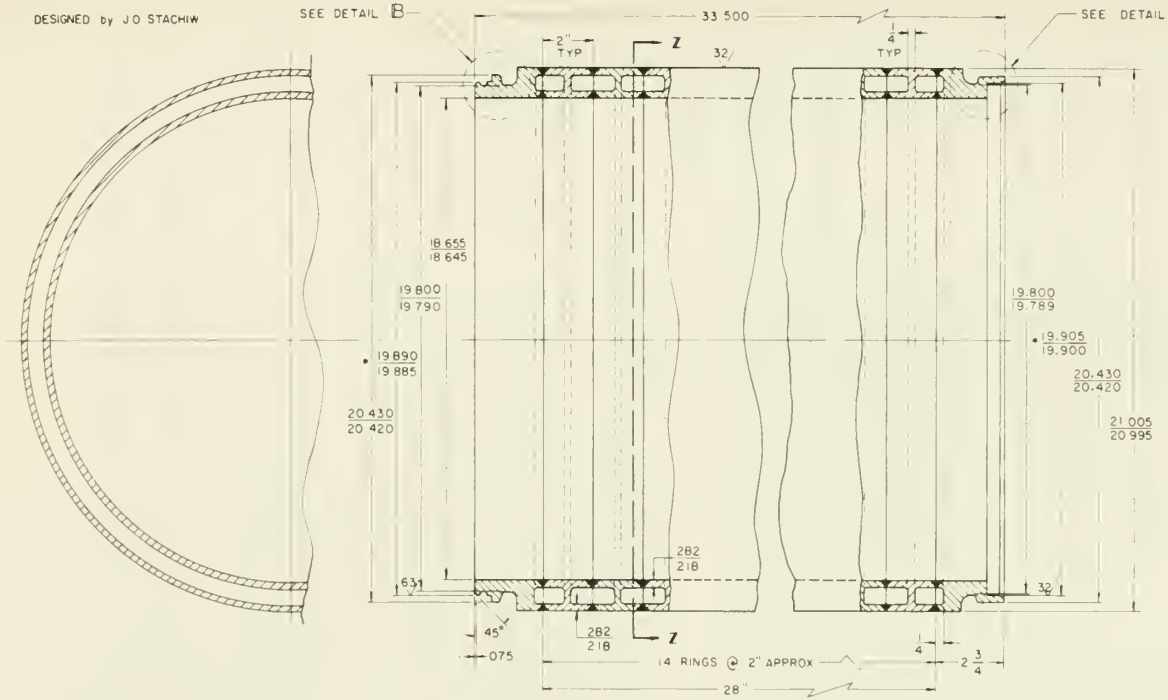
*To avoid reiteration, from this point on the circumferentially stiffened sandwich shells will be referred to simply as sandwich shells.

shells were very "tight." This was done to insure that the shells were as identical as possible so that they would collapse simultaneously during the implosion testing. The pertinent shell dimensions and description of the material properties can be seen in Fig. 9.

During the fabrication of the shells, all conceivable quality controls were instituted and adhered to in order to make certain that the final product tested was the same as described in Fig. 9. Both of the wrought aluminum shells demanded from the fabricator an unusually high degree of attention to manufacturing details, generally ignored in everyday shop practice. The welding fabrication method in particular presented more than just a usual share of fabrication headaches.

The wrought aluminum shells, due to the extreme length of welds and the required post-weld heat treatment, required special care to avoid any shell distortion caused by residual stresses. Only by using elaborate welding jigs and uniform welding rates was it possible to keep the distortion of the shells within the design specifications. The quality of the welds, the most important single item in the structural strength of the wrought aluminum shells, was so high that it even surpassed the fabrication specifications by 21 per cent, as shown in Table II.



NOTE:

1. DIAMETERS MARKED • TO BE CONCENTRIC WITHIN .005 TIR
2. AFTER WELDING, HEAT TREAT AND AGE TO "T6" TEMPER THE MATERIAL OF THE FINISHED SHELL SHALL HAVE THE FOLLOWING PROPERTIES TENSILE STRENGTH-42,000 PSI MIN, YIELD STRENGTH-35,000 PSI MIN, ELONGATION - 6% MIN IN TWO INCHES MATERIAL-6061-T6
3. MACHINED DIMENSIONS APPLICABLE AFTER HEAT TREATMENT
4. WELDS SHALL BE MADE USING FILLER ROD 4043
5. THE TENSILE STRENGTH OF WELDS SHALL BE 80 % OF MIN SPECIFIED TENSILE STRENGTH OF THE PARENT MATERIAL AFTER HEAT TREATMENT
6. WELDS MUST BE FREE FROM CRACKS, VOIDS, GAS POCKETS, SLAG INCLUSIONS, POROSITIES AND UNDERCUTS
7. THE FINISHED SHELL SHALL NOT LEAK AT 1100 PSI OF EXTERNAL PRESSURE THIS PRESSURE TESTING TO BE CONDUCTED BY ORL
8. VENT HOLES OF 1/16 DIA MAY BE DRILLED AS FOLLOWS: ONE HOLE IN EACH RIB AND ONE HOLE FROM WALL CAVITY TO INNER DIAMETER (ONE HOLE PER SHELL - THIS HOLE TO BE PLUGGED BY WELDING BEFORE MACHINING)
9. PRELIMINARY TO SHELL ASSY WELDING, TWO RINGS SHALL BE WELDED AND SUBMITTED TO ORL FOR WELD INSPECTION, TESTING, AND APPROVAL
10. "H" SHAPED RINGS MAY BE FORGED TO SHAPE SHOWN OR MACHINED TO SHAPE SHOWN FROM EITHER SOLID FORGED RINGS OR SOLID RINGS CONTINUOUSLY ROLLED FROM PIERCED WAFERS
11. THE EXTERIOR SURFACE, WHEN FINISHED, SHALL HAVE A HIGH POLISH WHEN POLISHING IS DONE, CARE SHOULD BE EXERCISED NOT TO BREAK SHARP EDGES
12. UNLESS OTHERWISE SPECIFIED:
    MACHINE FINISH 125
    BREAK SHARP EDGES .005 - .010 R
    CORNERS .005 - 010 R FILLET
    TOLERANCE ON SQUARENESS ± 15'
    DIAMETERS CONCENTRIC WITHIN .010 TIR

EXPERIMENTAL DESIGN
OF
WELDED ALUMINUM-ALLOY VESSEL
FOR
USE UNDER EXTERNAL PRESSURE
CELLULAR MODEL A & A'

FIGURE 9

A very important item in the design and fabrication of these shells was the location of the welds. Actually, there are several ways in which shell components may be joined together to form a welded shell structure, and the selection of the weld type and its location depends primarily upon the stresses that the designer thinks will be created by external pressure application. Since the external pressure loading generates the biggest stresses in the circumferential direction, it automatically eliminates the possibility of locating the welds along the shell's longitudinal axis. The only other possibility for locating the welds remained along the circumference of the shell. But even here there exist several alternatives for the selection of weld type and location.

The shell can be fabricated into a well-knit homogeneous unit only if the welds do not become the structural weak spots on the structure. Such a condition would have been created if the outside and inside facings were joined to the circumferential annular rings by slot welding. The slot welds, although sufficiently strong for ordinary structural use, would prove to be of insufficient strength for this application where both the facings and the ribs are loaded to their yield point in compression and the welded

slots must prevent the local separation of facings from the ribs at large shell wall deflections. This hypothesis has been proven by some local failures of circumferentially stiffened sandwich shells fabricated and tested by the David Taylor Model Basin.

Thus, upon critical evaluation of all the possible weld locations and types, a weld was selected that if properly applied would be almost as strong as the parent material, and yet due to its location would not become the weak spot of the shell. The welds (Fig. 9) have been placed in the circumferential direction joining the flanges of individual wide flange "I" rings which constitute the typical self-supporting shell sections.

Each of the two shells has been provided on each end with a wedge band joint for coupling with another shell of identical construction. The joints are equipped with standard neoprene O-rings in radial type sealing arrangement that effectively seal the shell assembly against high external testing pressures.

.2.2 TESTING APPARATUS

The apparatus required for the performance of implosion tests consists basically of two shell end closures, an

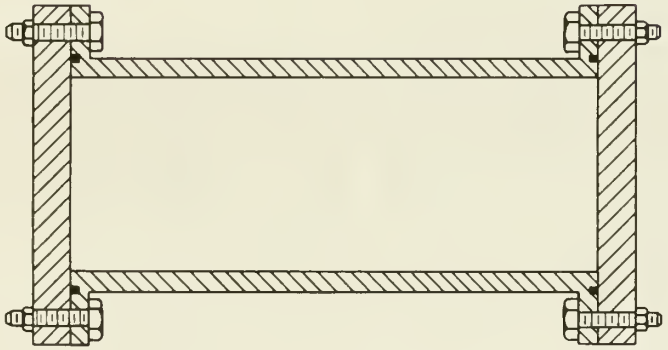
internal pressure vessel, a hydraulic pump, and several accurate pressure indicators. Pennsylvania State University, at the time this investigation was conducted, did not have pressurizing equipment of sufficient capacity to test the two aluminum experimental shells. After a short inquiry, pressurizing equipment of sufficient capacity and pressure capabilities was located at the Southwest Research Institute in San Antonio, Texas. Arrangements were made with the Southwest Research Institute, and it was in their facilities that all the experimental testing of aluminum shells described in this thesis was performed.

2.2.1 Discussion of Shell End Supports

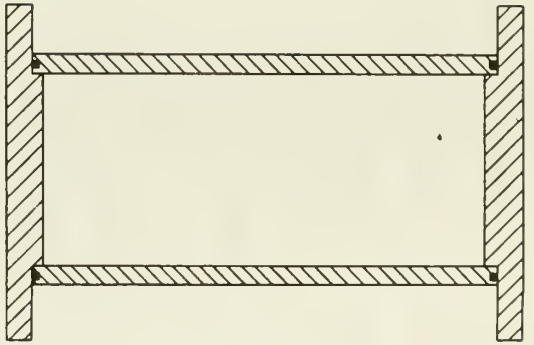
The primary consideration in the use of pressure tanks for implosion of shells is generally given to the method of mounting of the shells inside the tank. Depending on the type of shell support inside the tank, the experimental collapse pressures of any given shell will vary by a considerable order of magnitude, anywhere from 5 to 500 per cent, depending on the shell's h/D and L/D ratio between shell end supports.

The types of shell end supports can be classified into four broad categories: (1) rigid end supports,

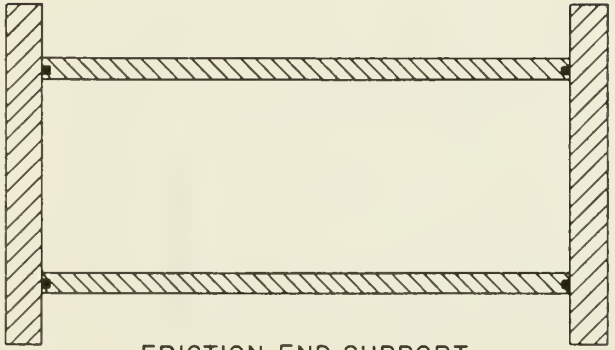
END SUPPORTS FOR VESSELS SUBJECTED TO EXTERNAL PRESSURE



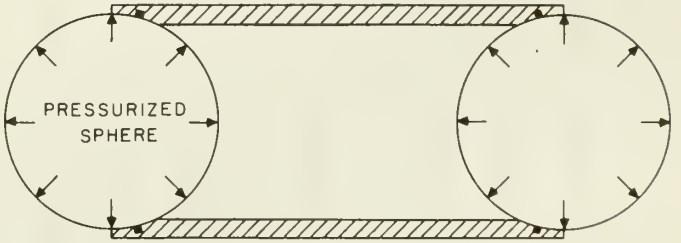
RIGID END SUPPORT



SIMPLE END SUPPORT



FRICTION END SUPPORT



ELASTIC END SUPPORT

FIGURE 10

(2) simple end supports, (3) friction end supports, and (4) elastic end supports (Fig. 10). Each type of end support imposes a different end condition on the shell which in turn may, and generally does, change the collapse strength of the shell. There is, generally speaking, no one preferred or best type of shell support; they all have their value depending on what the testing arrangement is supposed to reproduce or simulate. In this investigation, the friction type of end support was selected on the basis of the sponsor's requirements which specified that his interest lies in external pressure vessels of over five diameters in length and $.05 < h/D < .1$ thickness.

Shells whose dimensional parameters are such that a further increase in length will not change their collapse pressure are classified as infinitely long since their collapse pressure is independent of length[108]. Shells of interest to the sponsor, that is shells with $L/D > 5$ and $.05 < h/D < .1$, are considered to be infinitely long shells (Fig. 11c) for their collapse pressure is independent of length (t of smooth shells in Figs. 11a-11c is approximately equivalent to $\sqrt[3]{h^3-\left[h-(t_i+t_o)\right]^3}$ of circumferentially stiffened sandwich shells).

Equation (1), derived by R. von Mises[108], is shown in Figs. 11a-11c as a set of t/D ratio curves for the collapse of finite length tubes. This equation is used in this investigation only to prove that changing the spacing between rigid bulkheads influences drastically the collapse pressures of the tubes (smooth shells). The equation can not be used to predict accurately the collapse pressure of sandwich shells because, among other things, the equation does not take into account the change of modulus of elasticity with the stress level in the shell material.

To satisfy the sponsor's requirements, two approaches to shell testing are possible. First, the most straight forward and expensive approach requires experimental shells of over five diameters length rigidly or simply supported at the ends. Second, a less accurate but cheaper approach simulates the collapse strength of shells of five diameters length with much shorter shells equipped with friction end supports.

The reasoning behind the second approach is based on the fact that the collapse resistance (psi of external pressure) of a short shell ($L \leq 5D$ at $.05 < h/D < .1$) is actually the collapse resistance of an infinitely long shell stiffened to some extent by the presence of rigid

COLLAPSING PRESSURE OF TUBES SUBJECTED TO UNIFORM RADIAL AND AXIAL EXTERNAL PRESSURE

EQUATION BY R. VON MISES

$$y = \frac{1-\sigma^2}{n^2+\frac{\alpha^2}{2}-1}\left(\frac{\alpha^2}{\alpha^2+n^2}\right)^2 + \frac{X}{n^2+\frac{\alpha^2}{2}-1}\left[(n^2+\alpha^2)^2 - 2\mu_1 n^2 + \mu_2\right]\cdots$$

where

$$\mu_1 = 1/2\left[1+(1+\sigma)\rho\right]\left[2+(1-\sigma)\rho\right]$$

$$\mu_2 = (1-\sigma\rho)\left[1+(1+2\sigma)\rho-(1-\sigma^2)(1+\frac{1+\sigma}{1-\sigma}\rho)\rho^2\right] \qquad (1)$$

$$X = \frac{t^2}{3D^2}\ ;\ y = p\,\frac{D}{2t}\,\frac{1-\sigma^2}{E}\ ;\ \rho = \frac{\alpha^2}{n^2+\alpha^2}\ ;\ \alpha = \frac{\pi D}{2L}$$

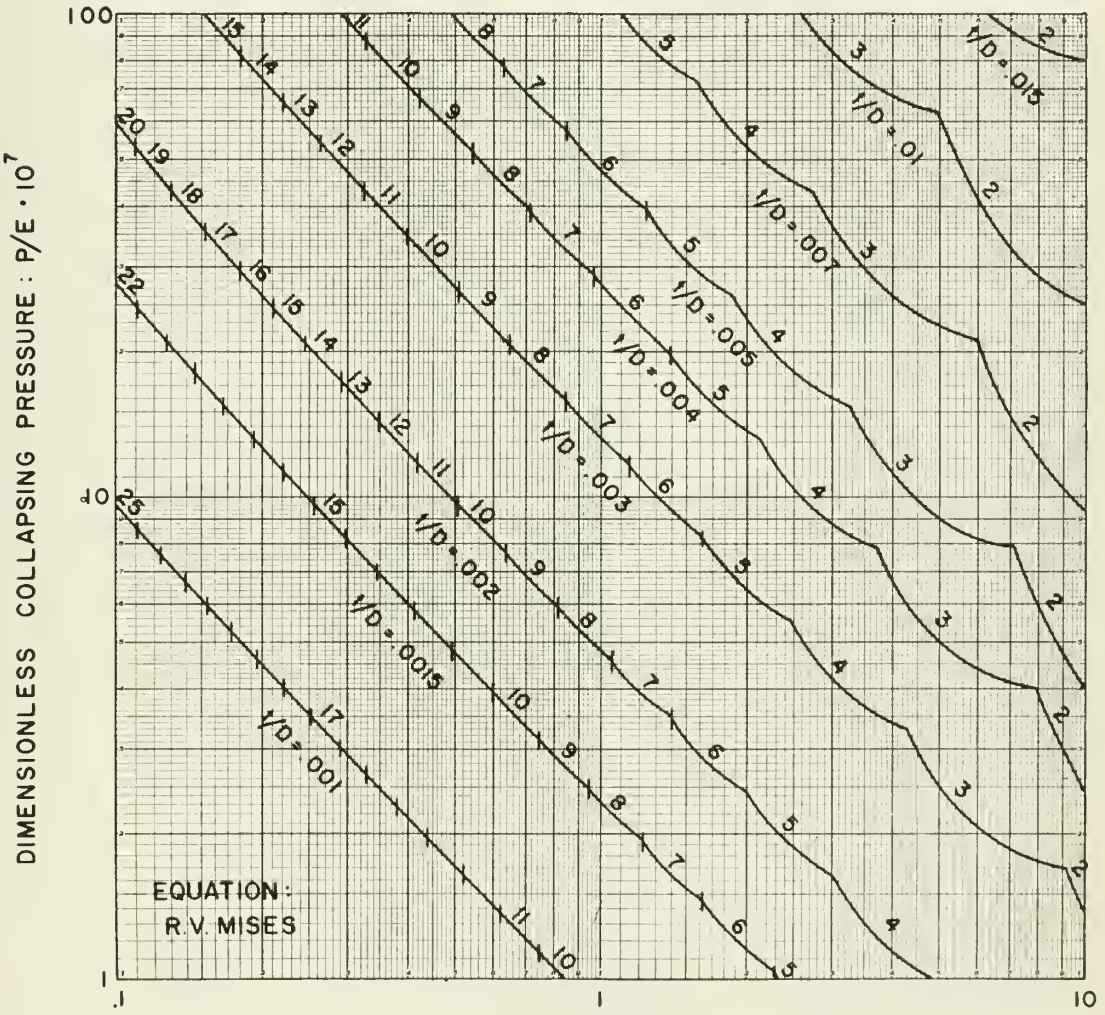
ASSUMED POISSONS RATIO = 0.3

FOR (n) THAT WHOLE NUMBER IS USED WHICH MAKES THE EXPRESSION FOR (p) A MINIMUM FOR A GIVEN D, (t), AND L

(Stodolas Festschrift, Zurich 1929 pp. 418-430)

COLLAPSING PRESSURE OF TUBES

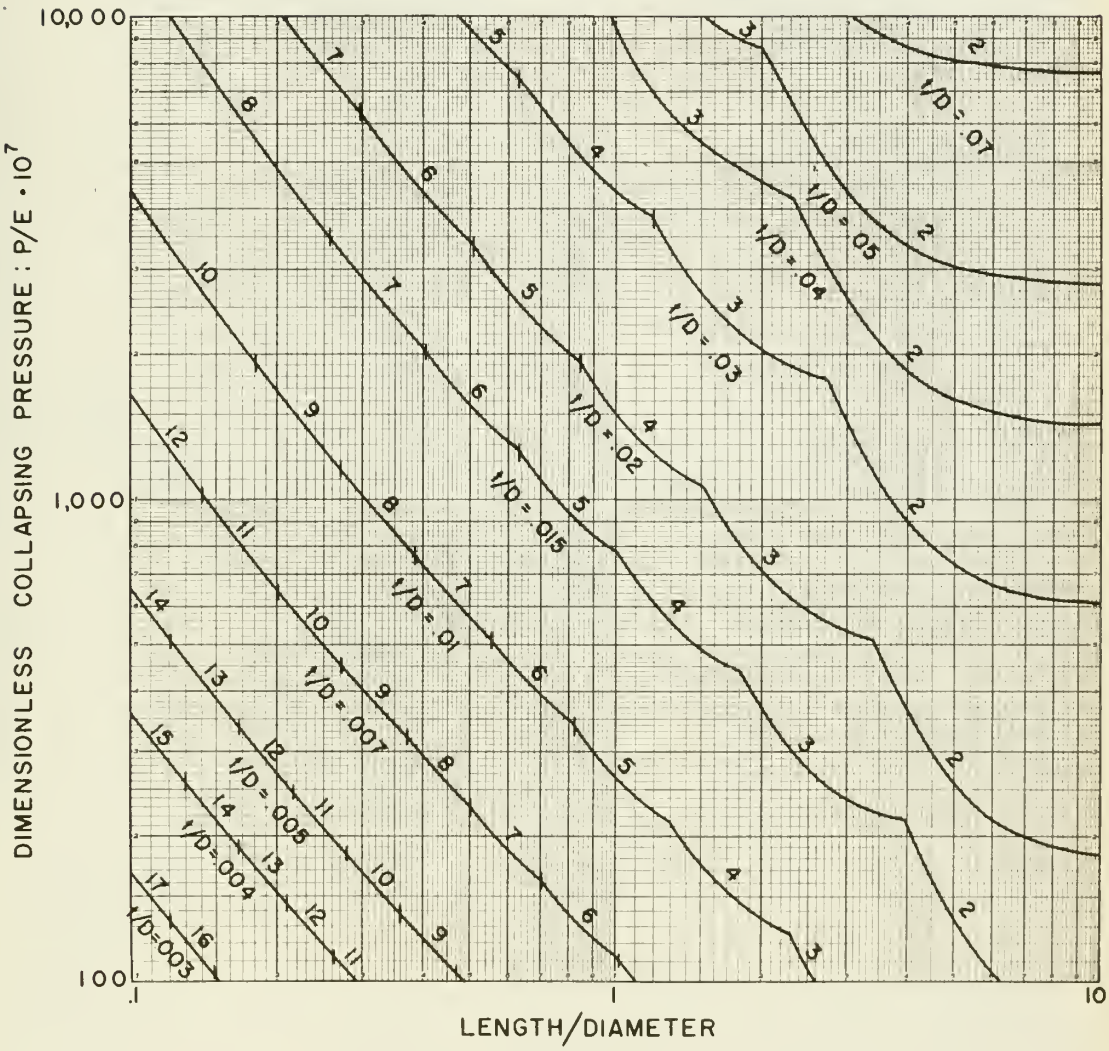
SUBJECTED TO UNIFORM RADIAL AND AXIAL EXTERNAL PRESSURE



PART I

FIGURE 11a

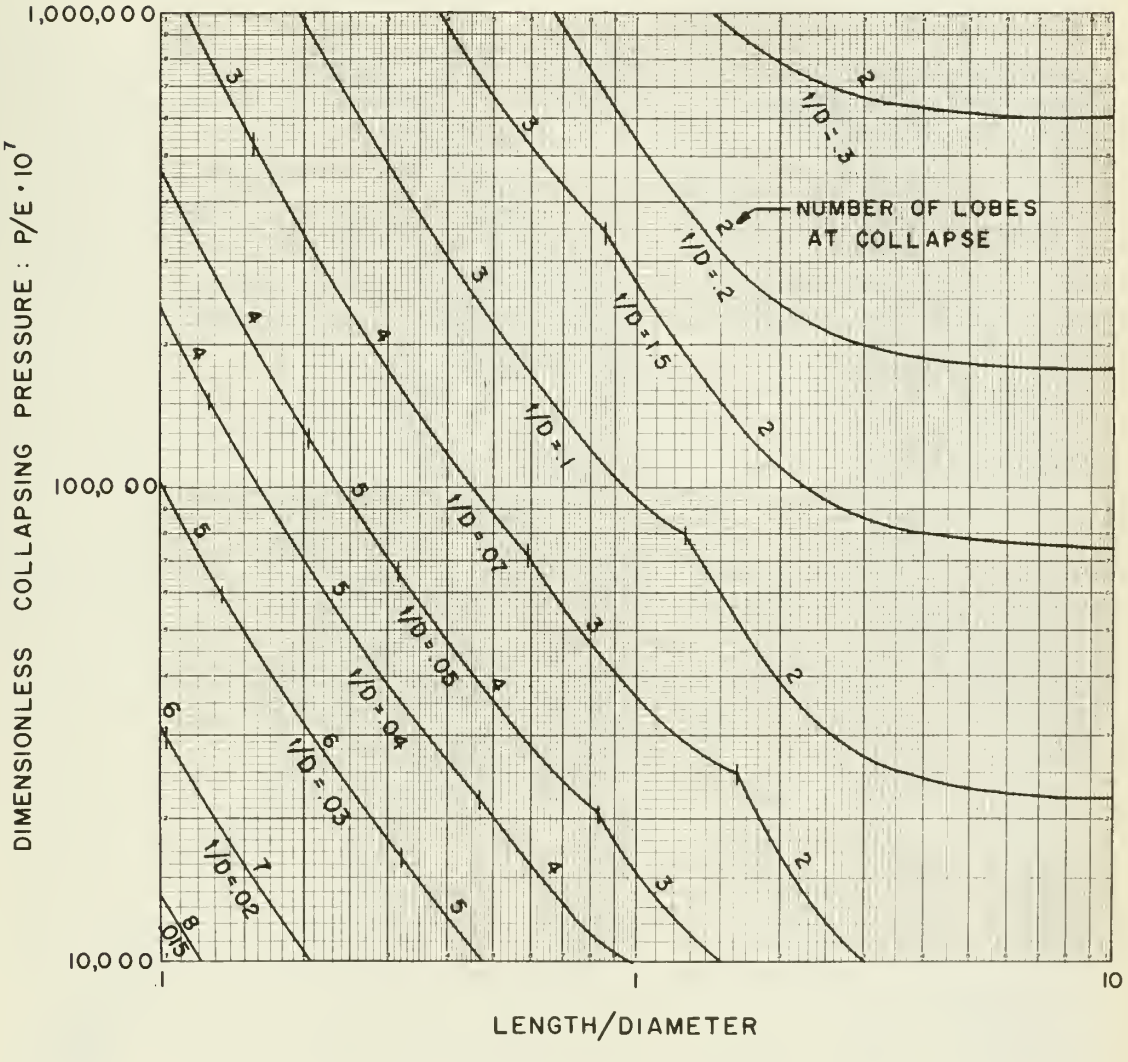
COLLAPSING PRESSURE OF TUBES
SUBJECTED TO UNIFORM RADIAL AND AXIAL EXTERNAL PRESSURE



PART II

FIGURE 11b

COLLAPSING PRESSURE OF TUBES
SUBJECTED TO UNIFORM RADIAL AND AXIAL EXTERNAL PRESSURE



PART III

FIGURE 11c

NUMBER OF LOBES INTO WHICH A TUBE WILL COLLAPSE WHEN SUBJECTED TO UNIFORM RADIAL AND AXIAL EXTERNAL PRESSURE

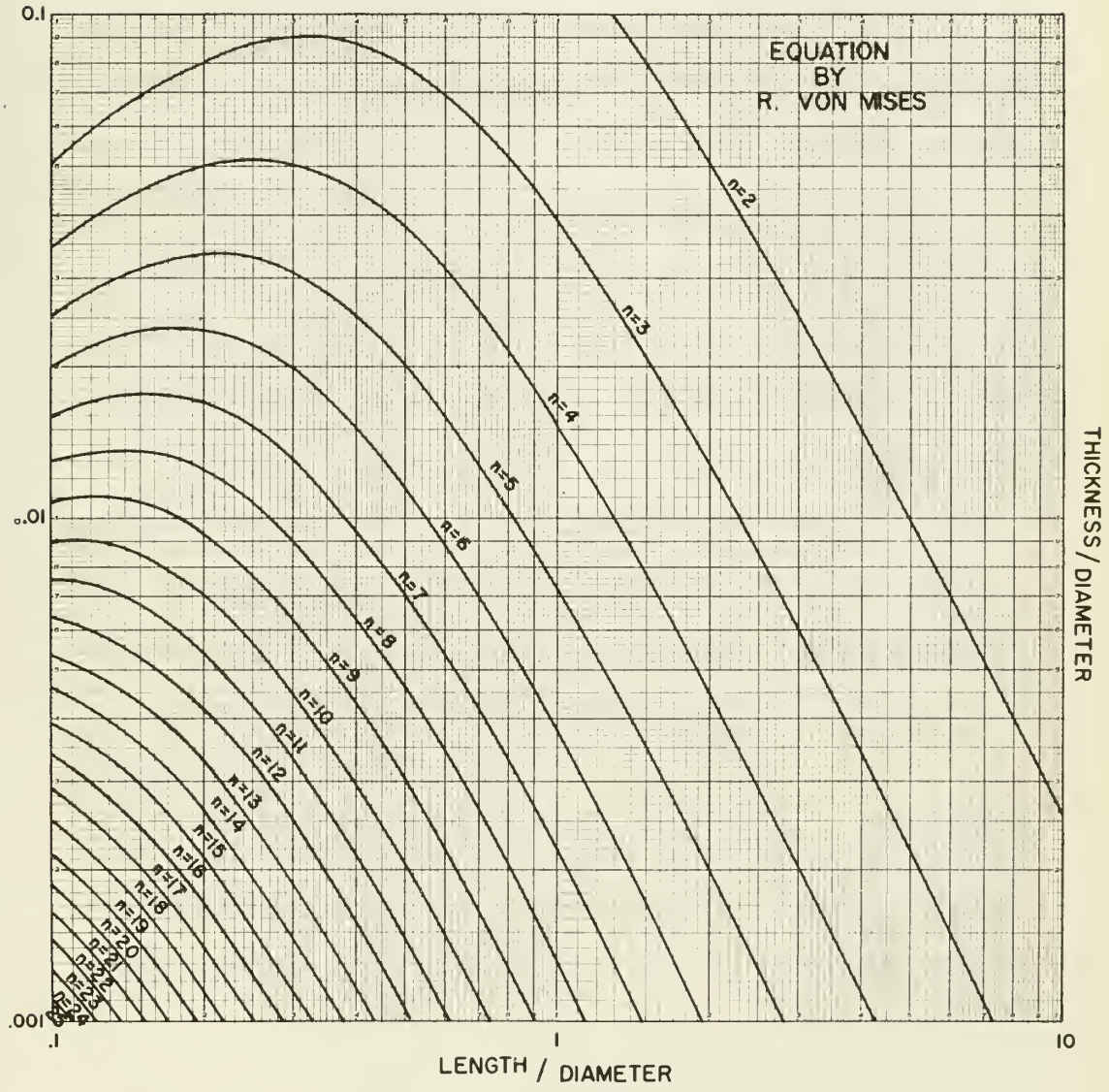


FIGURE 12

bulkheads at each end of the shell. This can be seen in Figs. 11a-11c where the collapse pressure of a $.04 < t/D < .09$ thick smooth shell (approximately equivalent in stiffness to a sandwich shell of $.05 < h/D < .1$ ratio) increases rapidly as the rigid bulkheads are brought closer together than L=5D. Thus if the stiffening action of the rigid bulkheads was actually known for the experimental implosion of a given short sandwich shell, then the collapse strength of an infinitely long sandwich shell of same h/D ratio could be obtained by simply subtracting the reinforcing action of the rigid bulkheads from the actual collapse pressure of the whole short sandwich shell assembly. Unfortunately, this is not an easy thing to do; the critical pressure graphs on Figs. 11a-11c have been experimentally proven only for smooth tubes while for sandwich shells they may be used only with the greatest caution, until more experimental evidence accumulates.

A reliable solution to this dilemna is to use instead of rigid bulkheads elastic end rings that slide upon closure plates while collapsing with the shell in a general instability mode. The stiffness of the end rings and the friction between the end rings and the closure plates is calculable and can be easily subtracted from the overall short sandwich shell

assembly collapse pressure giving, as an end result, the collapse pressure of a long sandwich shell. This type of end support has been used in this investigation both for the testing of the small scale acrylic resin shell series and for the testing of the large aluminum shells. The stiffness of the end rings and the friction between the rings and the closure plates were different for the two types of shells, but the variable parameters in each case were the same and could be calculated on the basis of the same equations.

2.2.2 Restraint Imposed by Friction End Supports

The friction end supports used in the investigation forming the basis of this thesis are not as good as the elastic sphere end supports (Fig. 10) whose rigidity does not vary from test to test and can be more precisely determined. The friction type of end support has been selected, however, due to its cheapness and ease of fabrication while still providing only minor rigidity increase to the shell section during the implosion test. The magnitude of the restraining action of this end support can be calculated with fair accuracy if the coefficient of friction between the shell and closure is accurately known. Since the coefficient of friction varies considerably with each alloy, actual surface

finish, and type of grease used, it is impossible to apply confidently the differing coefficients of friction quoted in the technical literature[124]. To obtain a static coefficient of friction in which confidence could be placed for the specific surfaces investigated, an experimental determination of static friction was performed using the inclined plane method. The static coefficient of friction between the end support plate and shell end adaptor ring, both fabricated from 6061-T6 aluminum with a $\sqrt{63}$ surface finish and lubricated with DC-4 silicone grease, was found to be $f_s=0.3$, while for DC-4 lubricated acrylic resin surfaces with $\sqrt{16}$ surface finish the coefficient was $f_s=.1$.

With the friction coefficients known, the restraining action of the closure plates could be calculated using accepted engineering equations.

If the column load is $N = \pi R_o^2 p_{ce}$ and the coefficient of static friction is denoted by f_s then the total end friction restraint is

$$M = f_s \pi R_o^2 p_{ce}$$

From this equation the radial restraining action of the

end friction can be shown to be

$$f_r = \frac{f_s \pi R_o^2 p_{ce}}{\pi R} = \frac{f_s R_o^2 p_{ce}}{R} \qquad (2)$$

where the units are lbs. per inch of shell circumference measured along the neutral wall axis.

The restraining action of the friction end supports becomes meaningful if it is expressed in terms of the increase of external collapse pressure due to the friction end restraint. The increase of the external collapse pressure due to the end restraint is dependent not only upon the collapse pressure itself but also upon the length of the shell between friction closures. The longer the shell, the less the end restraint due to friction will be felt along the length of the shell. In an infinitely long shell the end restraint effect on the collapse pressure becomes equal to zero. Thus, the coefficient of friction causes a definite increase in the experimental collapse pressure of a short shell; the shorter the section tested, the more pronounced the increase will be. Since economic factors limit the length of the shells, it becomes desirable to use as short a shell section as possible for the implosion testing. It is important, however, not to

shorten the shells to such an extent that the unpredictable variation of the friction coefficient during the application of external pressure will change the experimental collapse pressure by more than two or three per cent.

The collapse pressure increase of the short shells due to radial friction restraint at the shell ends can be expressed as:

$$\text{C. P. Increase} = \frac{\text{Radial Restraint at End Closures}}{\text{Length of Shell}}$$

where the collapse pressure increase is equal to the experimental collapse pressure of a short shell equipped with friction end supports minus the collapse pressure of an infinitely long shell. When written in engineering terms the equation becomes

$$\Delta p_{ce} = \frac{2 f_s R_o^2 p_{ce}}{RL} \qquad (3)$$

Equation (3) is limited only to shells of such range of dimensional parameters (L/D and h/D) where the collapse pressure is dependent upon the strength of the bulkheads. Figs. 11a-11c show whether a shell of given dimensional parameters is dependent on the bulkheads for its collapse

resistance. The range of L/D and h/D ratios for which Eq. (3) holds is shown as a series of arcs along inclined straight lines. Equation (3) does not apply to L/D and h/D ratios that lie on the horizontal line tangent to arcs indicating a bilobar mode of collapse.

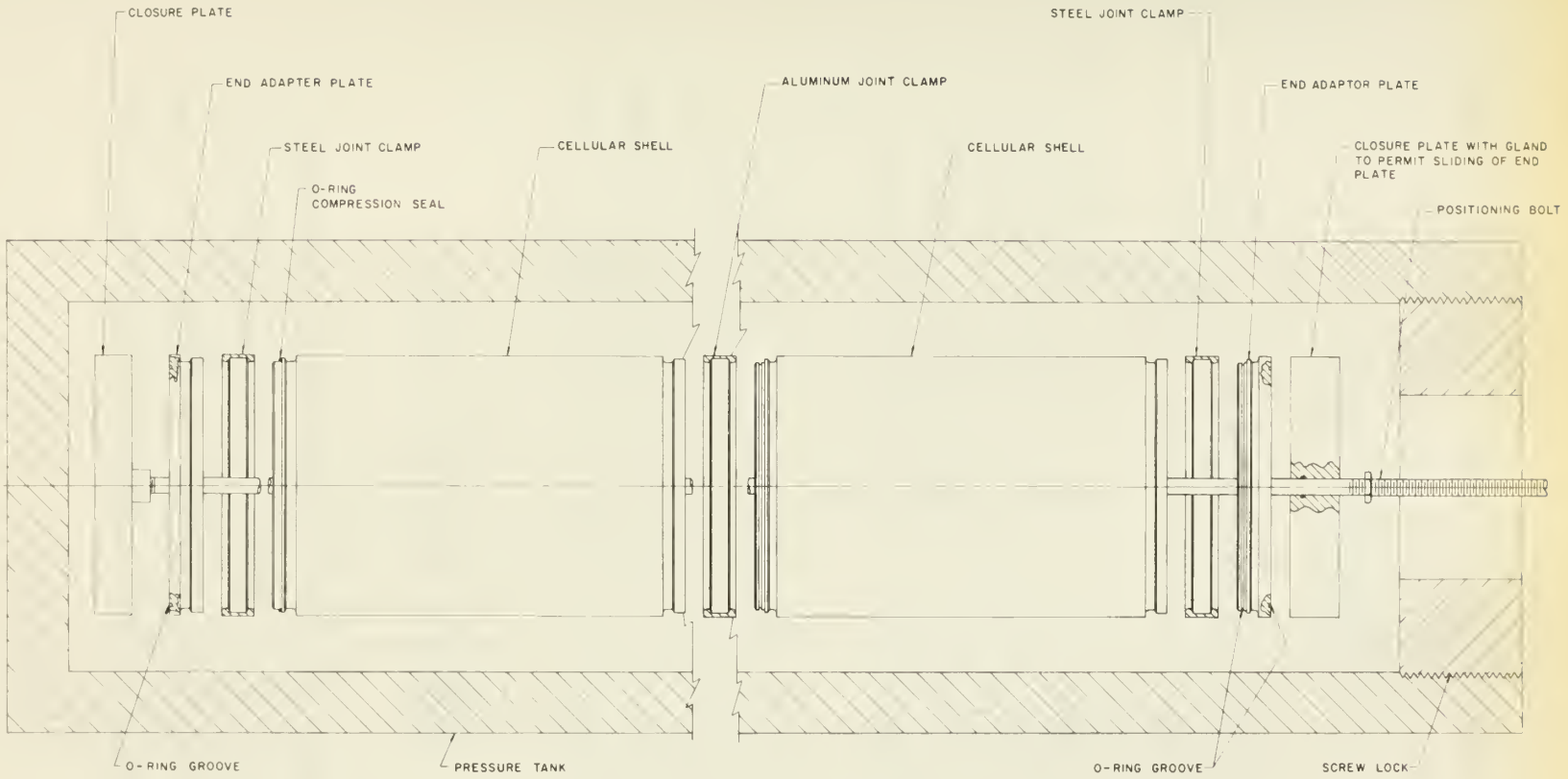
The vessels that have been tested to destruction in this general instability investigation were of such length and diameter (L/D = 3.5; h/D = .06 and L/D = 2; h/D = .14) that the derived Eq. (3) can be used with confidence for the calculation of experimental collapse pressure error due to the friction restraint at the ends.

2.2.3 Stiffening Action of End Rings

Besides the restraining effect of the friction end support, there is also the stiffening effect of the shell end adaptor rings and individual shell joints. The shell end adaptor rings, as can be seen in Fig. 13, are thin rings equipped with a wedge band joint on one end and an axial compression O-ring seal on the other end. The basic function of the shell end adaptor rings is to provide a transition stage from the shell proper to the friction end support. The individual shell joints, on the other hand, comprising also a larger mass of material, permit for testing purposes the joining of individual shells into one shell assembly.

The end adaptor rings permit testing of a pressure vessel assembly made up of several joined shells in a pressure tank equipped with friction end supports. The end adaptor rings can be eliminated if the shells are made with wedge band joints on one end for joining with other shells, and a compression type O-ring seal on the other end of the shell for sliding upon the friction end support closure plate. Fabrication of such shells is feasible but undesirable, for it deprives them of interchangeability with other shells. This lack of interchangeability would mostly be noticed when only one of the shells making up a pressure vessel assembly collapses under external pressure tests. The remaining shell could not then be tested to collapse by itself for it could not be made to seal against the end closure plates.

The stiffening action of the end adaptor rings and the individual joints can be calculated and the results show that the pressure increase due to their rigidity is quite small. The moment of inertia of the adaptor ring about its neutral axis in inches^4/inch of ring's axial length is only approximately 30 per cent larger than that of the typical "I" ring of the sandwich shells, Models A and A'. Depending on the length of the shell or shell assembly between the end adaptor rings, the increase in collapse pressure due to the



IMPLOSION TESTING
OF
EXPERIMENTAL PRESSURE VESSELS
AT
Southwest Research Institute
San Antonio, Texas

FIGURE 13

presence of these end rings can then be expressed as

$$\Delta p_{ce} = \frac{2(p_{ctr}-p_{cts})}{L} \qquad (4)$$

where p_{cts} = critical pressure of typical "I" ring

p_{ctr} = critical pressure of adaptor ring

This expression, like Eq. (3) for the collapse pressure increase due to closure plate friction effects, is reasonably true for only a limited region of L/D ratios, as shown in Figs. 11a-11c as a series of arcs forming inclined straight lines.

The expression for collapse pressure increases due to friction effect and the end ring stiffness effect can now be combined into one expression giving in psi the total collapse pressure increase caused by the testing apparatus. That is,

$$\Delta p_{ce} = \frac{2(p_{ctr}-p_{cts})}{L} + \frac{2f_s R_o^2 p_{ce}}{RL}$$

It is important to reiterate that Eq. (3) and Eq. (4) hold with reasonable accuracy only within a narrow band of L/D ratios. The increase of collapse pressure that it indicates is the theoretically predictable difference between true

collapse pressure of a given shell assembly without closure plates (infinitely long shell) and the actual collapse pressure of the same shell assembly equipped with friction type end closure plates and end adaptor rings.

2.2.4 Stiffening Action of Shell Joints

Besides the collapse pressure inaccuracies introduced by the presence of friction end supports and shell assembly end adaptor rings, there is also a discrepancy produced by the joints on individual shells themselves. The joint on each individual shell permits the joining of several short shells into one long pressure assembly for operational or testing purposes. The stiffening action of an individual shell joint is derived primarily from the bigger mass of material concentrated at the base of the joint.

The calculation of the stiffness of the individual shell joint is complicated considerably by the presence of the joint band that contributes to the stiffness of the joint assembly. In this investigation, it will be assumed that the rigidity of the individual shell joint is made up of the sum of rigidities of the joint on the shell and of the joint band around it. The increase in the collapse

pressure of the individual shell due to the stiffening effect of individual shell joints can be calculated using the expression:

$$\Delta p_{ce} = \frac{2(p_{ctj}-p_{cts})}{L} \qquad (5)$$

Summing up all the restraints acting on the aluminum shell assembly, consisting of Models A and A' joined together, the pressure increase during implosion testing due to the action of all the restraints can be expressed as

$$\Delta p_{ce} = \frac{2f_s R_o^2 p_{ce}}{RL} + \frac{2(p_{ctr}-p_{cts})}{L} + \frac{4(p_{ctj}-p_{cts})}{L}$$

where f_s = static friction coefficient

p_{ctr} = theoretical collapse pressure of a shell end adaptor ring

p_{ctj} = theoretical collapse pressure of a shell joint

p_{cts} = theoretical collapse pressure of a typical shell "I" ring

p_{ce} = experimental collapse pressure of the shell assembly

L = bulkhead spacing

R_o = external radius of the shell

R = radius of the neutral wall axis

The predicted collapse pressure increase due to all the conditions discussed above holds only for failure by general instability and does not apply in the slightest manner to local buckling or local yielding or fracture of the material from which the shell is made. The proof for this can be deduced readily from the proposition that only in cases of general instability does the reinforcing action of the shell ends extend over the whole shell span[106]; in all other failure types, the buckling, yielding, or fracture is of localized character over which the distant shell ends have no influence. This limitation is important as otherwise local collapse pressures may be corrected for end effects and give completely meaningless collapse values. In cases of doubt, a general good criterion to apply is that only in cases of general instability do the individual shell joints and end rings of a short shell assembly buckle with the rest of the shell assembly.

2.3 TEST FACILITIES AT THE SOUTHWEST RESEARCH INSTITUTE

2.3.1 Pressurization System

The pressure tank (Fig. 13) in which the implosion testing of the shells was conducted, is situated in the

Mechanics Laboratory of the Southwest Research Institute located at San Antonio, Texas. The tank, although not the largest, possesses pressure capabilities that are not surpassed by any other tank of such dimensions in the nation. The approximate useful dimensions of the tank are 29" inside diameter by 150" long and it is able to contain safely pressures up to 10,000 psi. Its construction (see Fig. 13) is simple and yet very reliable. The tank is actually composed of a section of a straight thick tube threaded internally at both ends and capped with solid steel discs. The sealing between the discs and the tube is accomplished by standard O-rings backed with steel expansion rings. The cap on the loading end of the tube has an 8" diameter opening that permits observation of the inside of the shell during implosion testing. One may add here that although observation of the inside of the shell during the test is possible, it is not permitted due to safety reasons unless a remote control viewing apparatus is used, such as closed circuit television or a robot camera. The whole tank assembly is positioned inside a concrete lined silo in the floor of the building with the loading end of the tank being almost flush with the floor.

The pressurizing medium at Southwest Research Institute is heavy motor oil (SAE 30) that has been found by experience preferable to tap water. The lubricating, rust-proofing and dielectric qualities make the use of oil very desirable since it eliminated extra work in the performance of a pressure test. The dielectric qualities of the oil, in particular, eliminate the need for the waterproofing of electric resistance strain gages that are often mounted on the outside of the shells tested.

An unusual feature of this pressure tank installation is the use of air driven pumps for the pressurization of the oil inside the tank. The air driven pump is a very compact unit that does not require elaborate starting and overload relays as does an electrically driven unit of similar size and capacity. The air fed to the pump is of relatively low pressure (approximately 150 psi) and does not present any special hazards to the operator of the pump. The pump control and the pressure gage are located on a single panel permitting the pressurizing operation of the tank by only one person.

2.3.2 Instrumentation

To indicate and record the strains and deflections of the shell subjected to external pressure inside the

pressure chamber, several types of instrumentation are available at the Southwest Research Institute. To mention but a few, one can point to the electric resistance gages and the hydraulic shell deflection indicator. For this particular investigation, the electric resistance strain gages and strain recording equipment were used exclusively.

For the recording of strains indicated by SR-4 gages, an automatic scanner-recorder was used that permitted the balancing and recording of 48 gage circuits in one minute. The rapidity with which all the strains could be read and recorded eliminated any discrepancy between different gage readings at a given pressure due to creep of the shell material or glue with which the gages were attached to the shell.

The instrumentation on the shell consisted of electrical resistance strain gages mounted on critical areas of the shell assembly. The strain gages, single element 1/4" paper backed wire-wound gages, were arranged on the shell into two element 90° rosettes whose axes coincided with the shell's axial and circumferential planes. The location and identification of gages can be seen in Figs. 14 and 15. Since all of the strain gages were mounted on the inside of the

shell, it was necessary only to have a temperature compensating gage and omit the pressure compensating gage.

2.4 TESTING PROCEDURE OF SHELL MODELS A AND A'

Both the instrumentation and testing procedures were planned in such a manner as to provide the greatest amount of information possible with the limited funds allotted for this purpose by the sponsor. Besides getting experimental collapse pressure of Models A and A', it was also desirable to obtain information about the influence of end conditions on the collapse pressure.

The twin shells, Models A and A', were assembled into one pressure vessel assembly capped at both ends with the friction type end closure plates (Fig. 13). After placement inside the pressure chamber, the chamber cover was screwed down tight and the entrapped air in the chamber bled off to the atmosphere. After checking for any signs of leakage, the oil inside the chamber was pressurized to 100 psi and all the strain gages zeroed in at that pressure. The pressurizing of the oil in the tank and the recording of strains were performed simultaneously by two operators, the pump operator following orders from the strain recorder

INSTRUMENTATION DURING EXTERNAL PRESSURE CYCLING TEST

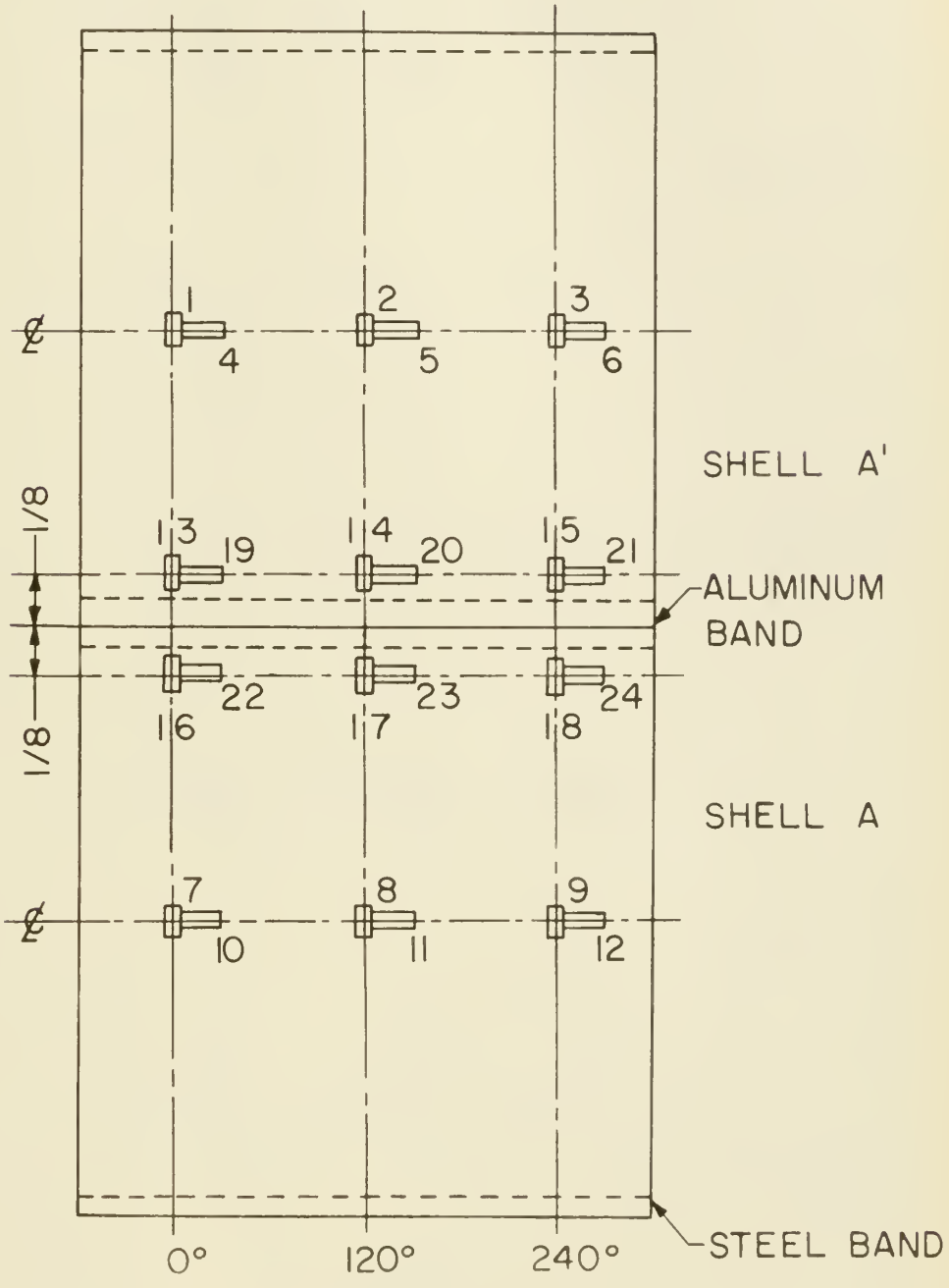


FIGURE 14

INSTRUMENTATION DURING IMPLOSION TEST

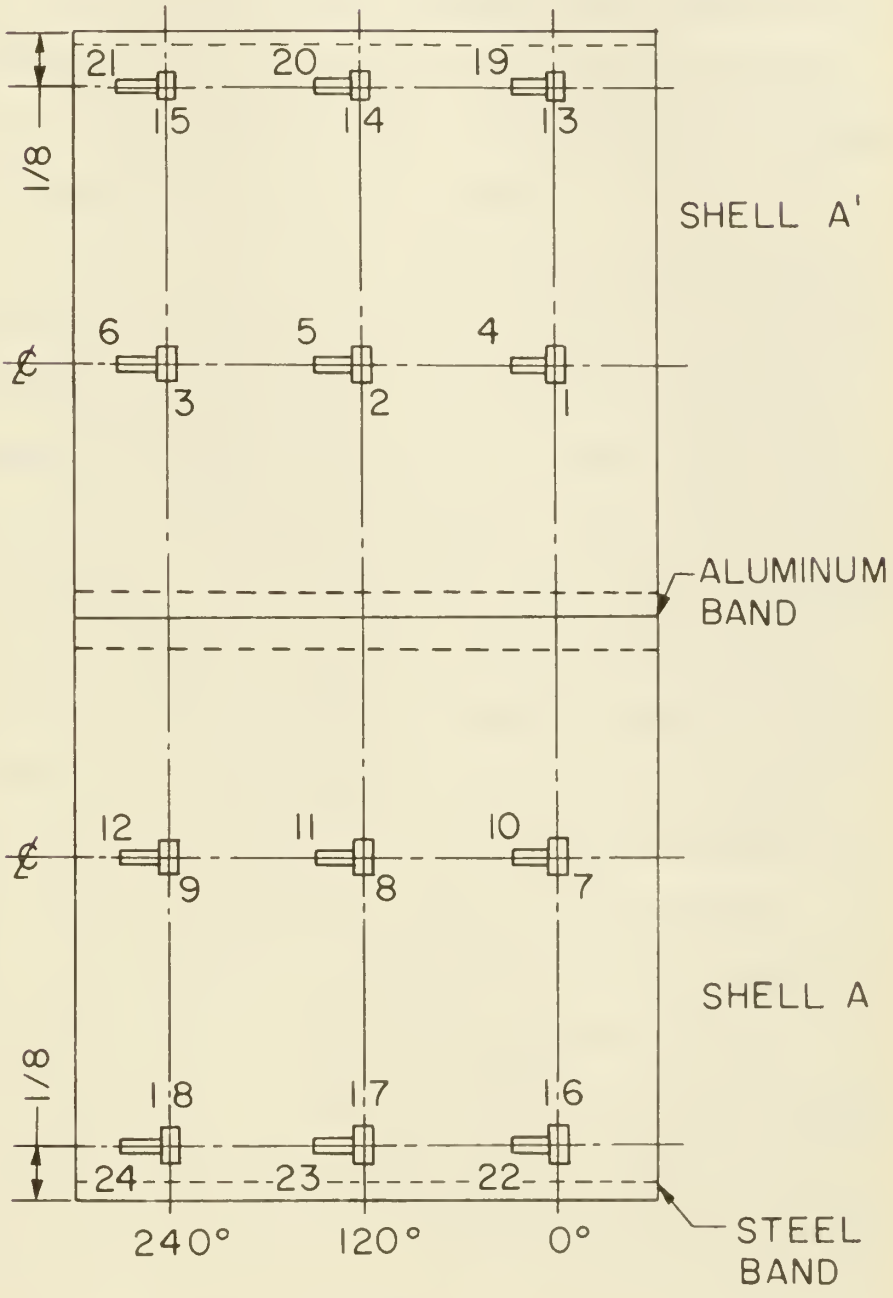


FIGURE 15

operator. Upon command the pressure was increased 100 psi and held there for the duration of the strain recording cycle by the automatic scanner-recorder. Upon completion of the recording cycle, the pressure was raised again 100 psi and the recording cycle repeated. This procedure was repeated until a pressure of 1100 psi was reached.

At 1100 psi the strain recorder was disconnected and the pressure cycled twenty five times from zero to 1100 psi. The cycling of pressure at this pressure level (1100 psi) eliminated any residual stresses in the shell assembly due to the prior welding and heat-treating. After completion of the cycling, the strains were recorded again in an identical manner to check for any creep or redistribution of strains that may have occurred during the repeated pressure cycling. Upon completion of the pressure cycling stress relieving program, the shells were taken out of the tank, coupled in reverse order and again positioned inside the pressure chamber. This operation was performed for several reasons, none of them pertinent to this study except for the re-positioning of shells in order to obtain some strain readings at the shell assembly ends resting against the end closure plates. In such a manner the circumferential strain readings,

obtained both when the gages were in the center and at the ends of the shell assembly, could be compared to each other to detect the influence of the end adaptor rings sliding upon the end closure plates.

Having performed all the necessary preliminary residual strain relieving and gage zeroing, the shell assembly was ready for the implosion test. The pressure was raised now in 200 psi intervals and the strains recorded at each pressure interval level. The pressure increases were continued until implosion of the shell assembly occurred at 2300 psi. Both of the shells in the assembly collapsed simultaneously so that any further testing of a single shell was not necessary.

Upon removal from the pressure chamber, the shells were measured for distortion (Fig. 23) and dissected (Figs. 24 and 26). Both the thickness of "I"-ring flanges and of webs were compared to the drawing specifications to determine any possible deviations from the fabrication specifications (Fig. 9). Since the welds comprise a large amount of filler material on the shell, coupons were cut from the imploded shells and tested to destruction to determine the strength of the welds.

Only upon completion of the above mentioned tests were the experimentally obtained data cross referenced and

conclusions drawn about the validity of the sandwich shell general instability theory expressed in Eq. (8).

CHAPTER III

EXPERIMENTAL RESULTS

3.1 SHELL STRAINS UNDER EXTERNAL PRESSURE

The strains given by the electrical resistance strain gages (Figs. 16-22) give a wealth of information about the behavior of the shells under load. The information obtained from them has been used in this investigation to obtain partial answers to the following specific four questions:

1. Were there any residual stresses and strains in the shells prior to the pressure test?
2. Did the individual shell ends increase the resistance to collapse of individual shells?
3. Did the friction type shell assembly end supports and end adaptor rings increase the collapse pressure, as theoretically postulated?
4. Was the buckling resistance of both shells of the same magnitude?

Comparing the strain record of mid-bay gages taken at the beginning and at the end of the cycling test, no difference could be found between them, and so only one set

of curves is plotted (Figs. 16-19). This indicates that there was possibly only a negligible amount of residual stresses present, as otherwise, the difference between the strains recorded at the beginning and at the end of the test would have been noticeable, indicating that some realigning of stresses had taken place. This realigning was expected to occur because of the repeated elastic loading and unloading of the shell structure. However, the results do not, apparently, bear this out.

There exists a considerable difference between the strains at shell mid-bays and at the shell ends, proving that individual shell ends are stiffer than the "I" rings. That the individual ends increase the overall rigidity of the shells is obvious from the collected data but the actual increase cannot be deduced from the data. Even though the difference in magnitude of circumferential strains between the ends and the shell mid-bys can be calculated, this difference is not directly proportional to the stiffening action that the individual shell ends exert on the shell. This can be explained by the fact that the circumferential strains at any point are directly proportional to the wall cross-section but not to the moment of inertia of the shell

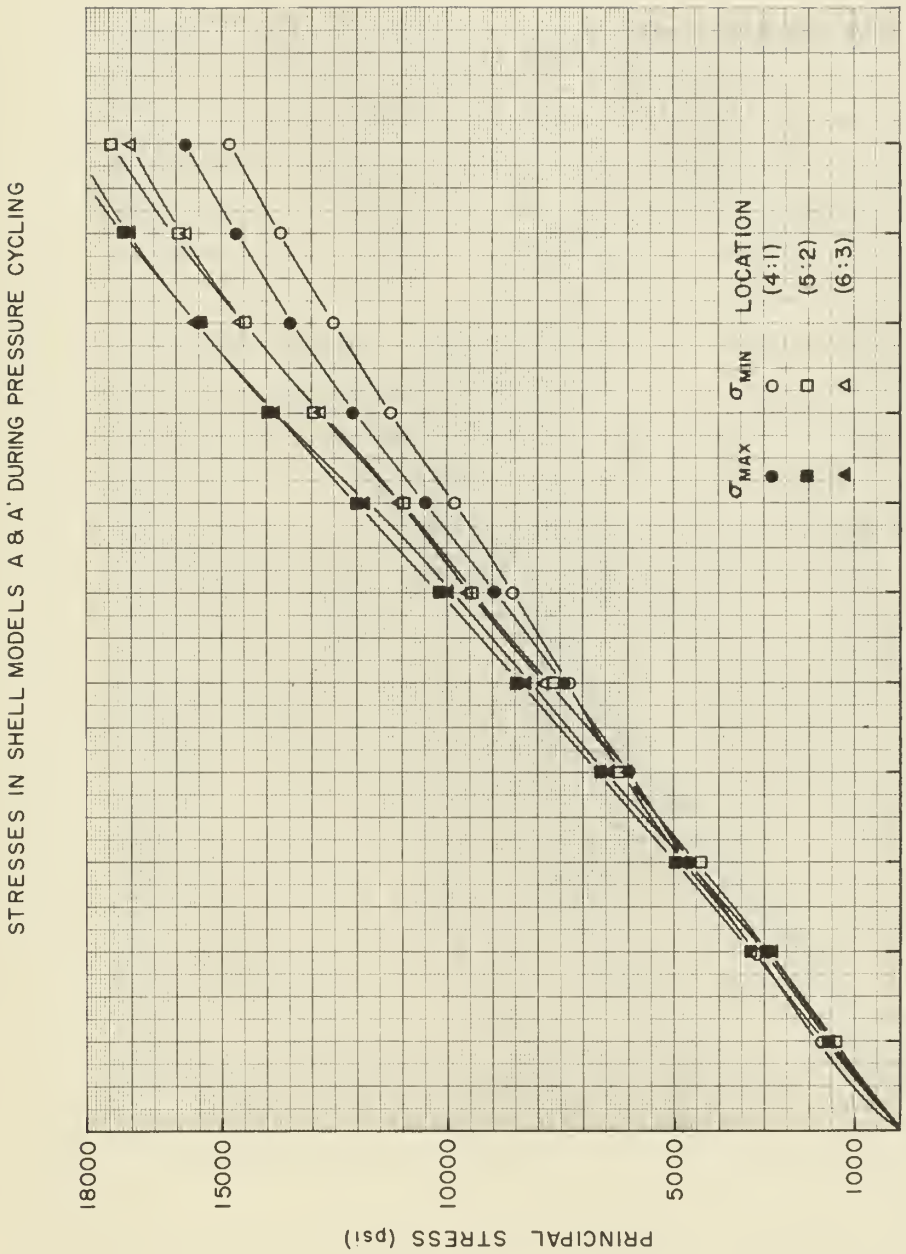

STRESSES IN SHELL MODELS A & A' DURING PRESSURE CYCLING
PRINCIPAL STRESS (psi)
18000
15000
10000
5000
1000
0
100
200
300
400
500
600
700
800
900
1000
1100
EXTERNAL PRESSURE (psi)
σMAX σMIN LOCATION
(4:1)
(5:2)
(6:3)

FIGURE 16

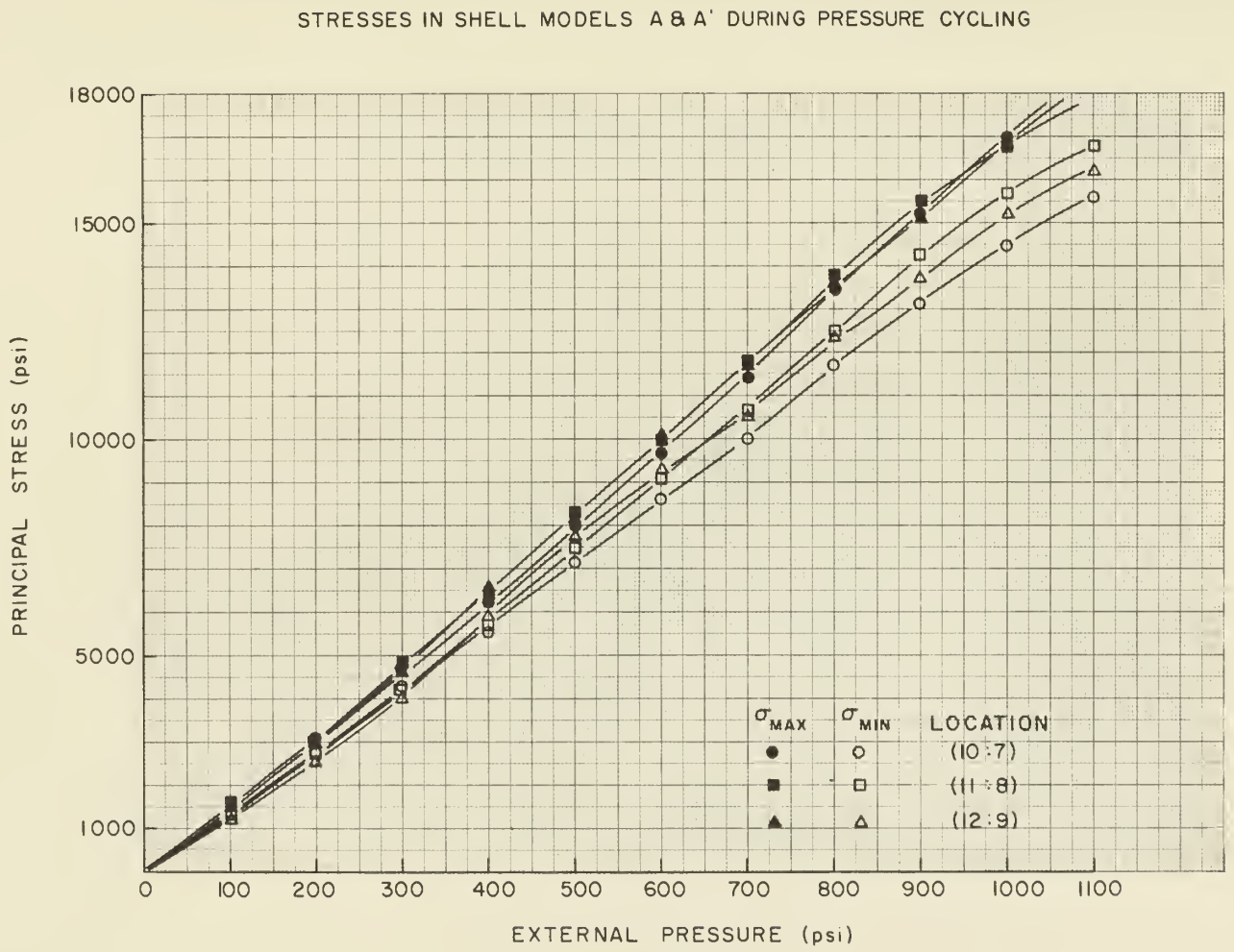

STRESSES IN SHELL MODELS A & A' DURING PRESSURE CYCLING
PRINCIPAL STRESS (psi)
18000
15000
10000
5000
1000
0
100
200
300
400
500
600
700
800
900
1000
1100
EXTERNAL PRESSURE (psi)
σMAX
σMIN
LOCATION
(10:7)
(11:8)
(12:9)

FIGURE 17

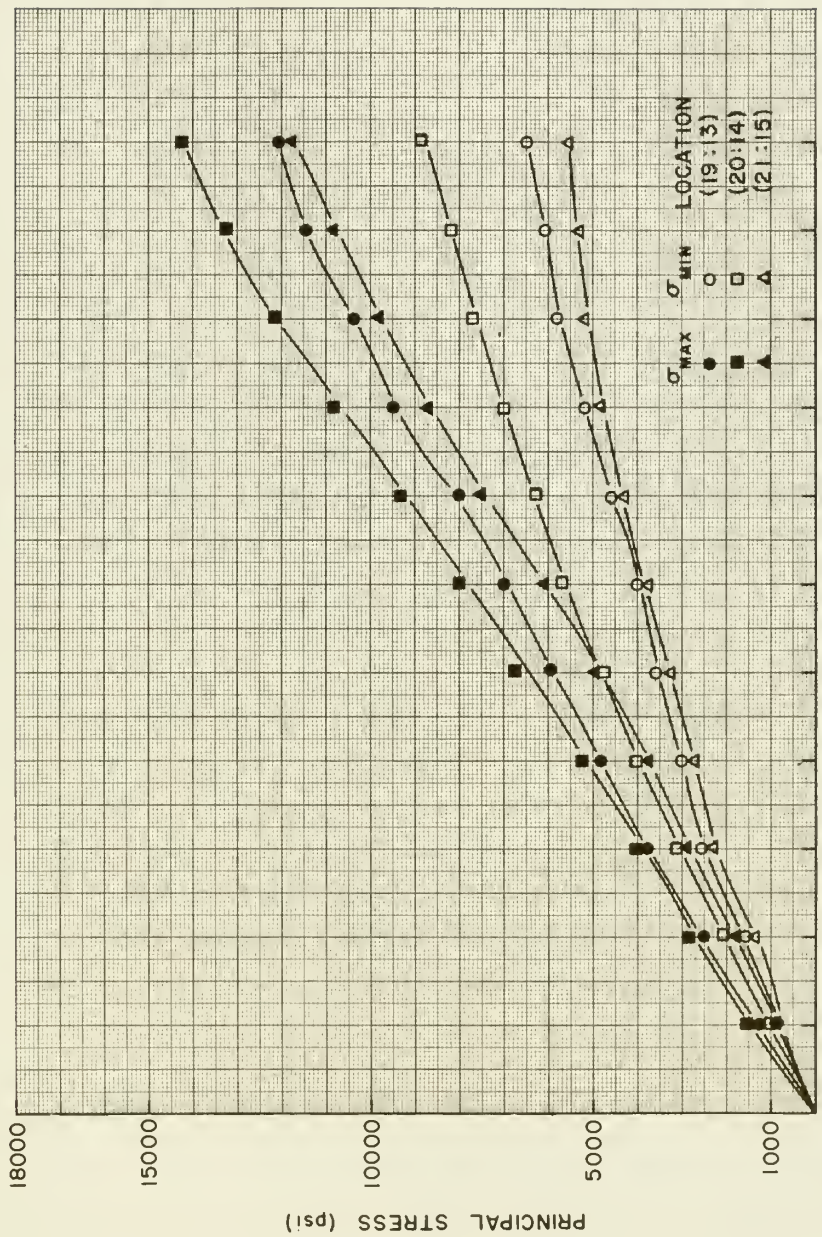

STRESSES IN SHELL MODELS A & A' DURING PRESSURE CYCLING
PRINCIPAL STRESS (psi)
18000
15000
10000
5000
1000
0
100
200
300
400
500
600
700
800
900
1000
1100
EXTERNAL PRESSURE (psi)
σ MAX σ MIN LOCATION
(19:13)
(20:14)
(21:15)

FIGURE 18

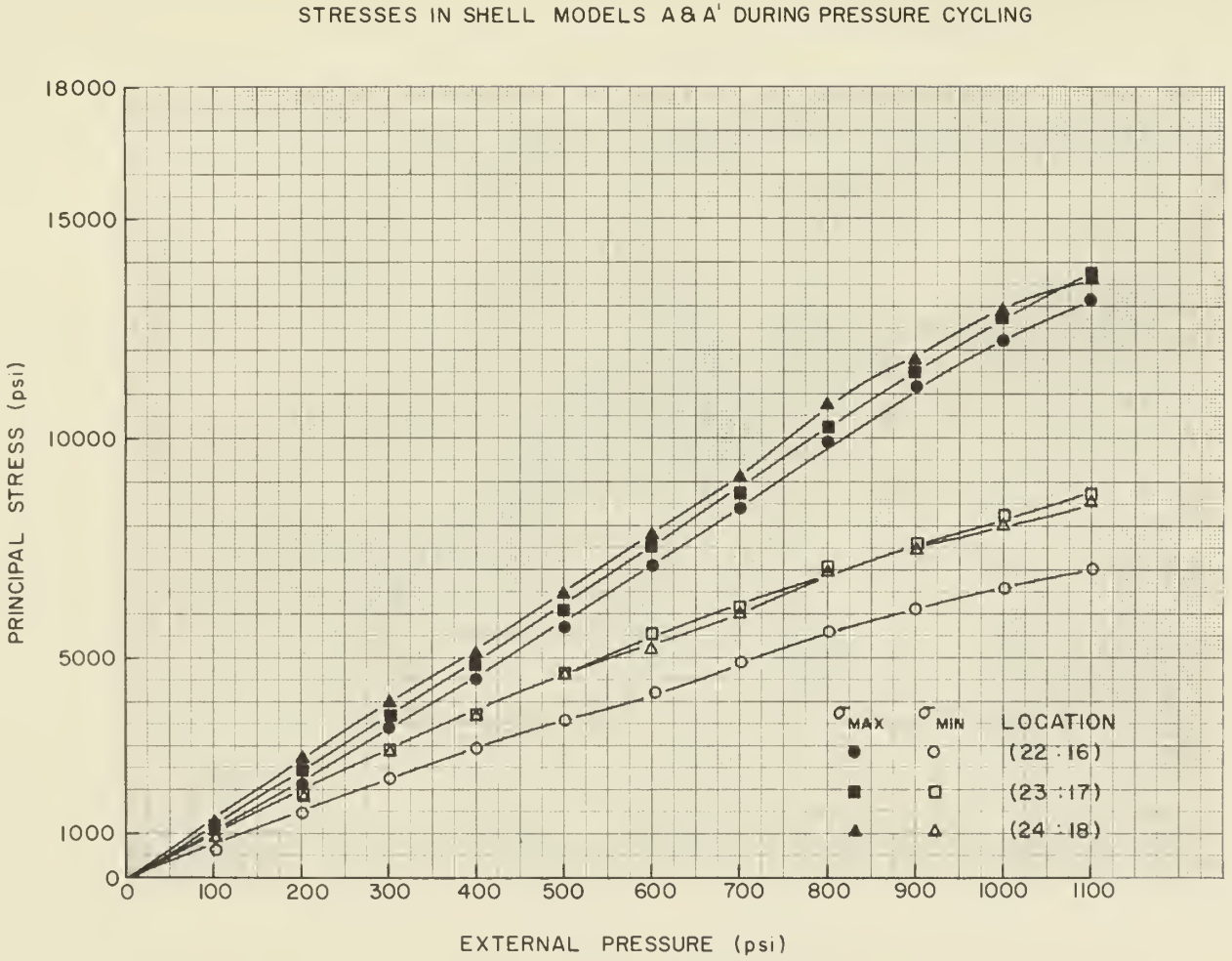

STRESSES IN SHELL MODELS A & A' DURING PRESSURE CYCLING
PRINCIPAL STRESS (psi)
18000
15000
10000
5000
1000
0
0
100
200
300
400
500
600
700
800
900
1000
1100
EXTERNAL PRESSURE (psi)
σMAX
σMIN
LOCATION
(22 : 16)
(23 : 17)
(24 : 18)

FIGURE 19

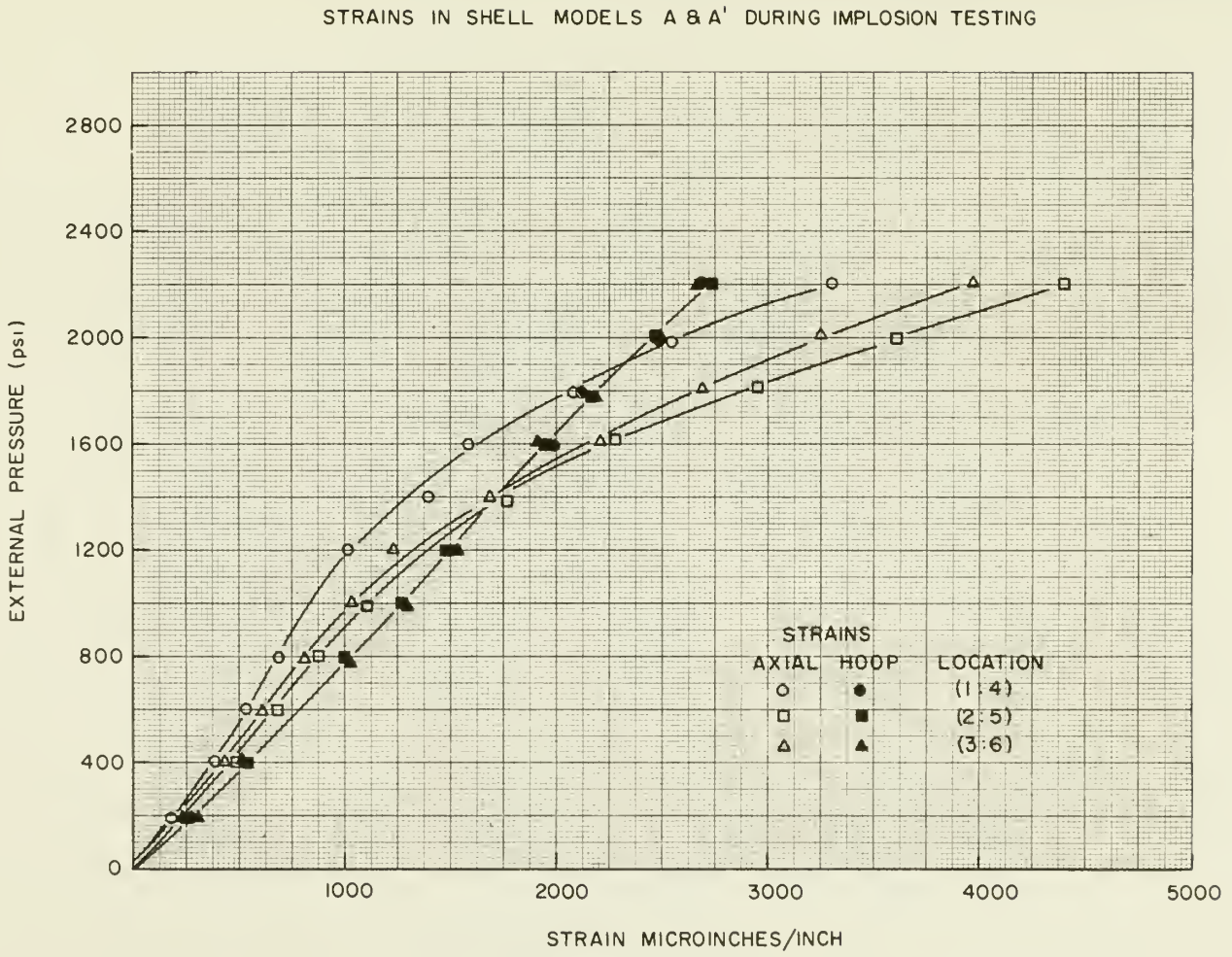

STRAINS IN SHELL MODELS A & A' DURING IMPLOSION TESTING
EXTERNAL PRESSURE (psi)
2800
2400
2000
1600
1200
800
400
0
1000
2000
3000
4000
5000
STRAIN MICROINCHES/INCH
STRAINS
AXIAL HOOP LOCATION
(1:4)
(2:5)
(3:6)

FIGURE 20

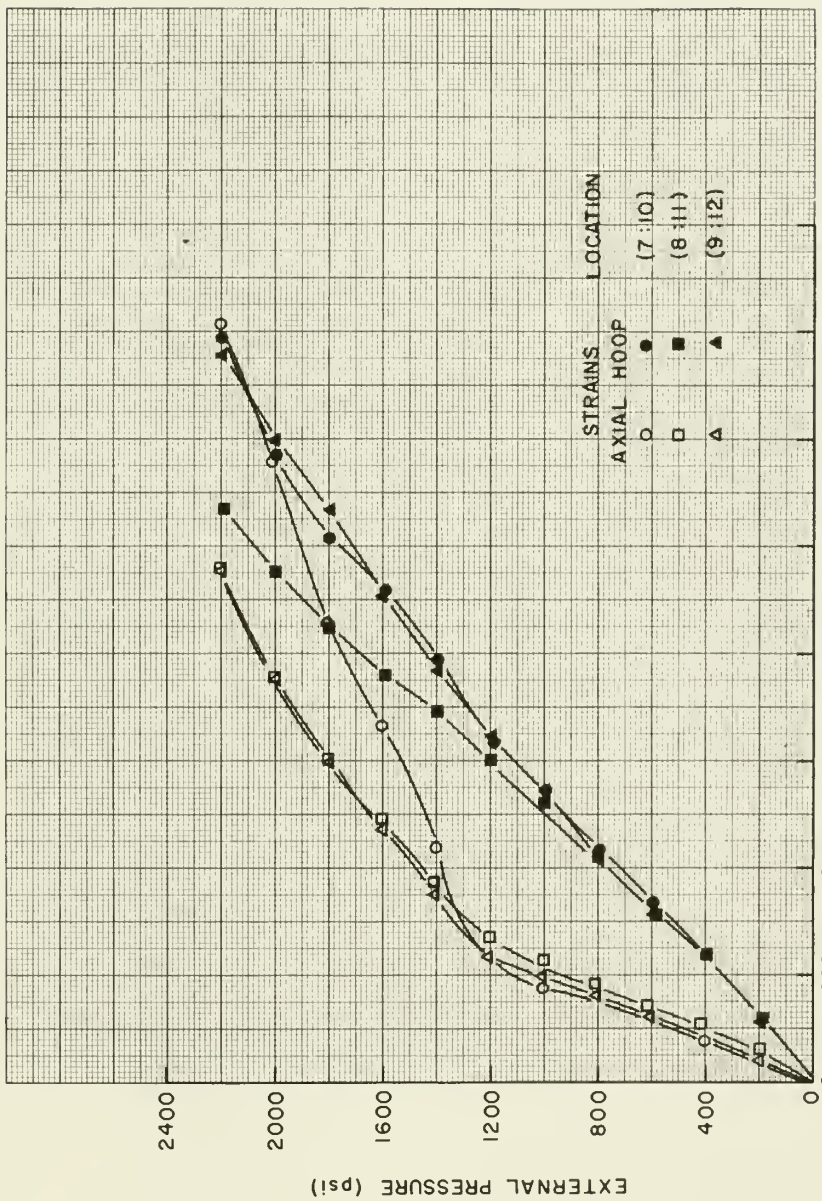

STRAINS IN SHELL MODELS A & A' DURING IMPLOSION TESTING
EXTERNAL PRESSURE (psi)
0
400
800
1200
1600
2000
2400
STRAIN MICROINCHES/INCH
0
1000
2000
3000
4000
5000
6000
7000
8000
STRAINS LOCATION
AXIAL HOOP
(7:10)
(8:11)
(9:12)

FIGURE 21

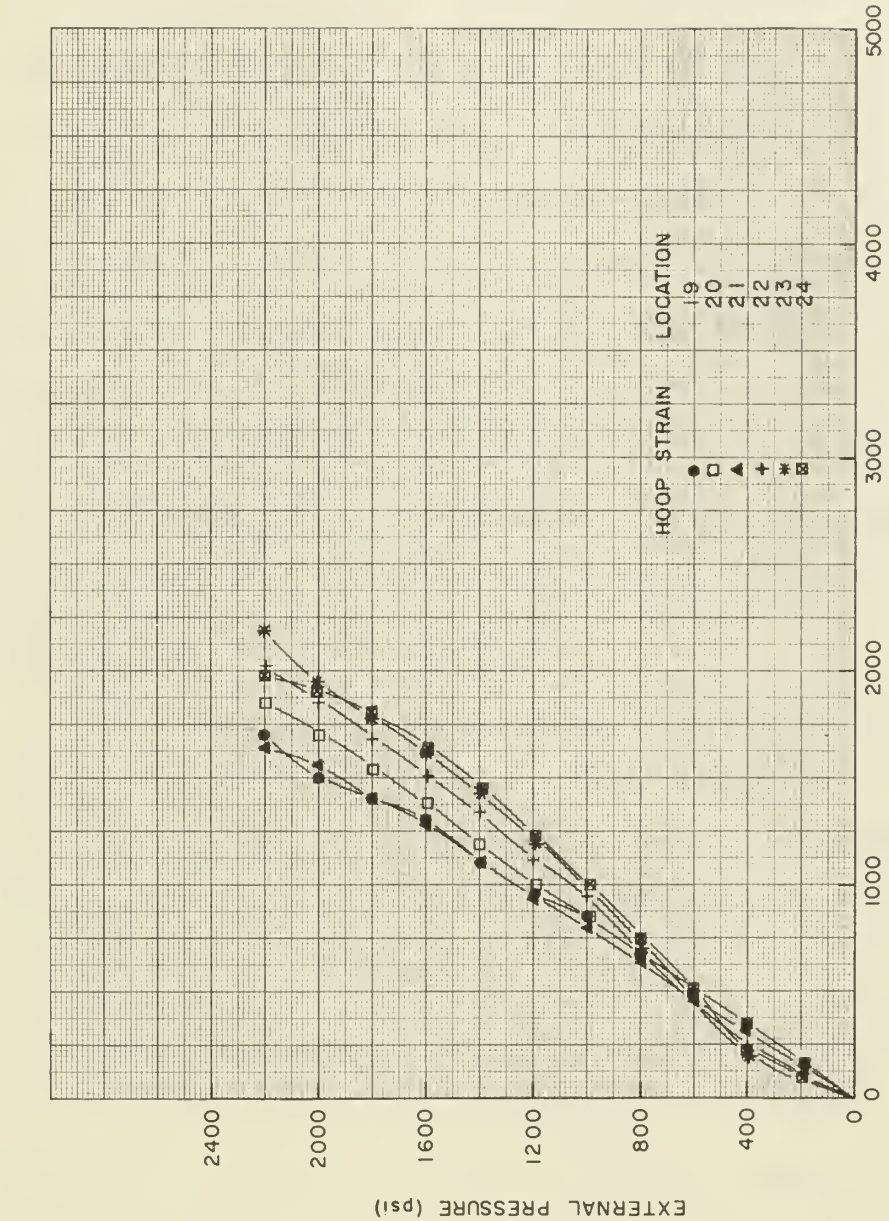

STRAINS IN SHELL MODELS A & A' DURING IMPLOSION TESTING
EXTERNAL PRESSURE (psi)
0
400
800
1200
1600
2000
2400
STRAIN - MICROINCHES/INCH
0
1000
2000
3000
4000
5000
HOOP STRAIN LOCATION
19
20
21
22
23
24

FIGURE 22

wall at that location. Since the shells collapsed in a general instability mode, their collapse resistance was directly proportional to the moment of inertia of the "I" rings, and not necessarily to their cross-sections. Thus, although the circumferential strains at the individual shell ends were 50 per cent less than those at mid-bay, the rigidity and, thus, resistance to collapse of the shell ends may be only 20 to 30 per cent larger than that of a typical "I" ring section. To this, one may add the fact that the lower circumferential strains at individual shell ends act only over a very short distance (approximately 1") so that their total influence over a shell of length L, as it has been shown in Eq. (5), is only slight.

Both of the shells tested imploded simultaneously but their rate of deformation and extent of damage is not the same. The strain gages at mid-bay locations do not indicate any noticeable difference between circumferential strains of Models A and A' but the gages located at the ends of the shells show a difference. When the circumferential strains at the ends of the two shells are compared, it becomes apparent that the ends deformed at different rates and thus supplied different amounts of restraint to the shells causing one to fail sooner than the other. The difference in the

final amount of deformation that took place can be deduced from the measurements taken of the outside shell diameters at different points along the length of the shell (Fig. 23). Judging by the difference in the plastic deformation of the two shells, one can venture an engineering guess that the collapse strengths of the two shells differed probably by 50 to 100 psi, which is less than 5 per cent of the actual experimental collapse pressure of 2300 psi.

3.2 DISSECTION OF FAILED SHELLS

The visual observation of the whole collapsed shell (Fig. 25) as well as the dissected shell (Fig. 24) shows that there are no indications that any local buckling occurred before the total collapse by general instability. Detailed observation of the "I" ring flanges and webs proved that all of these members were in excellent shape. Such a result is of great importance for it eliminates from any further considerations the need to discuss the influence of local instability on the buckling by general instability. The fact that local buckling did not occur can be pointed out to be the single most important experimental item found in this investigation for it lays a secure foundation

PERMANENT DEFORMATION OF THE SHELLS AFTER IMPLOSION

X IN	O.D. MIN.	O.D. MAX.
0	18:44	22.90
5	18:21	22:79
10	18:23	22:80
15	18:23	22:78
20	18:40	22:51
25	18:50	22:47
29 1/4	19:01	22:32
0	19:41	22:34
5	19:41	21:85
10	19:82	21:60
15	19:92	21:35
20	20:22	21:10
25	20:40	21:00
29 1/4	20:51	20:90

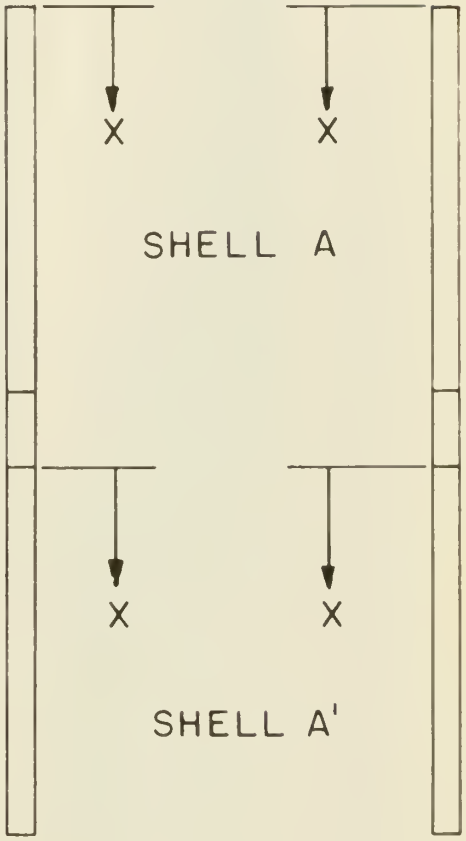


FIGURE 23

FIGURE 24

DISSECTED COLLAPSED SHELL, MODEL A

FIGURE 25

SHELL MODEL A AFTER IMPLOSION

TABLE II

MECHANICAL MATERIAL PROPERTIES OF THE COLLAPSED SHELLS

MODELS A AND A'

Test Description:

Parent Material: 6061-T6 Aluminum Alloy
Weld Filler: 4043 Sigma
Weld Type: 90° Single Vee Butt Weld
Testing Method: Tensile
Strain Rates: .001"/sec.

Mechanical Properties:

	Coupon	Cross-Section	Elongation* in 1"	Load lbs.	Ultimate Strength
Parent Material		0.0986	17.47	4765	48,327
Parent Material		0.095	19.50	4650	48,947
Weld Coupon	1	0.1005	4.42	4010	39,900
Weld Coupon	2	0.1003	3.90	4005	39,930
Weld Coupon	3	0.0995	5.18	4200	42,211
Weld Coupon	4	0.102	4.06	4110	40,294
Weld Coupon	5	0.0997	3.68	3910	39,218
Weld Coupon	6	0.1003	4.60	4045	40,329
Weld Coupon	7	0.101	4.08	3855	38,168
Weld Coupon	8	0.1005	4.46	4260	42,388
Weld Coupon	9	0.1005	4.89	4180	41,592
Weld Coupon	10	0.1005	4.89	4165	41,443

* percent

Average Weld Strength = 40,547 psi

Average Parent Material Strength = 48,637

Average Weld Strength = 83.5 per cent of parent material strength

Average Weld Strength = 121 per cent of specified weld strength

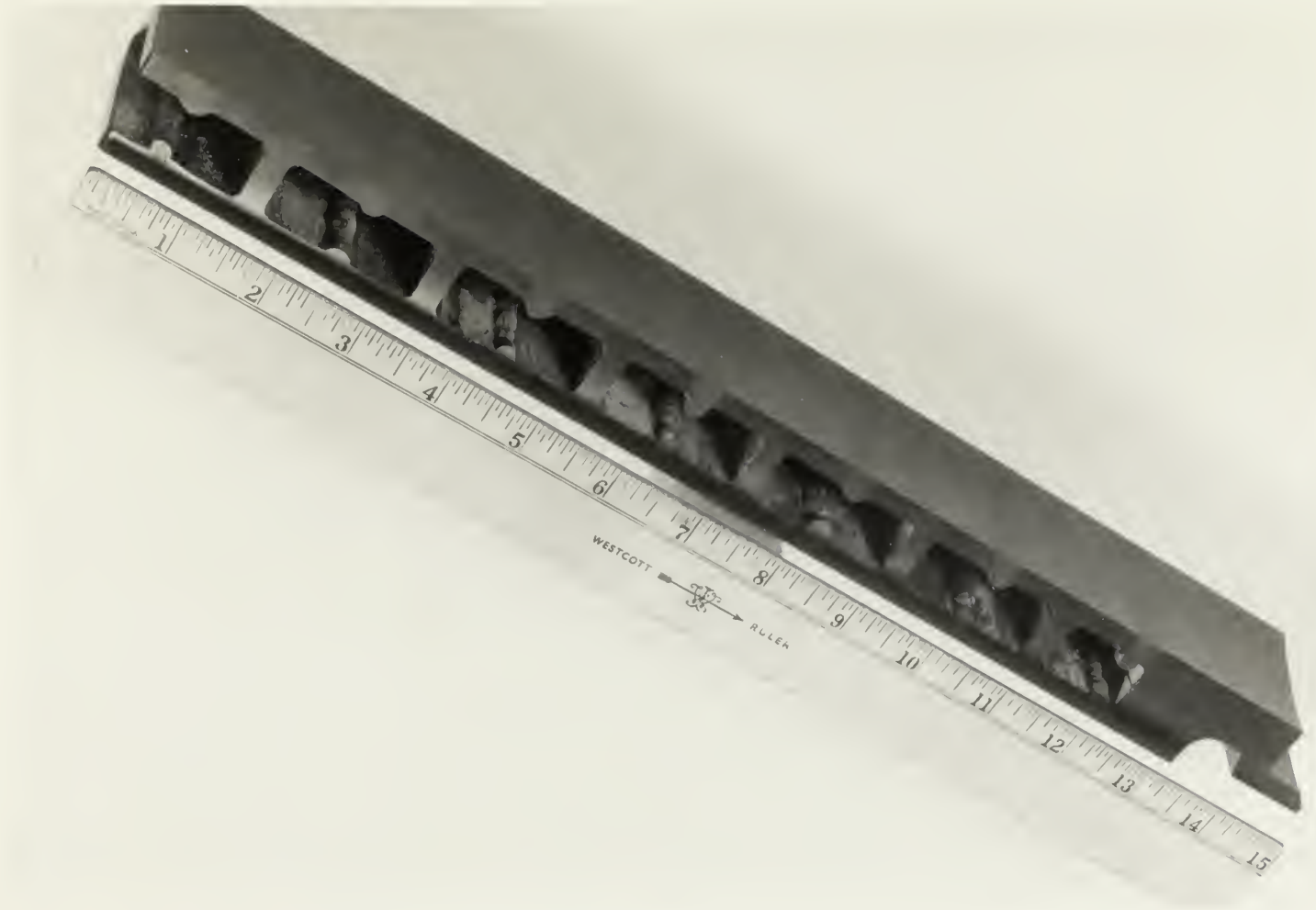


FIGURE 26

SANDWICH SHELL WALL SECTION

for the comparison of theoretical and experimental shell collapse pressures due to general instability. If local buckling were present, the comparison between theoretically predicted and experimentally determined collapse pressures would be impossible. The theory developed in this thesis presupposes only the existence of general instability untainted by the influence of local failures due to local buckling or material fracture.

To determine whether the shells actually represent the shell specified on the drawing, accurate measurements at various locations were taken of the "I" ring flanges, webs and web spacings. All the measurements taken failed to disclose any deviation from the specifications called for in Fig. 9. To find out more about the shell test(Table II), coupons were cut at various locations and subjected to tensile tests in a hydraulic press. Both the material and the welds were found to surpass the engineering specifications by approximately 20 per cent. After the completion of these "post mortem" tests, one must admit that he is confronted with an example of excellent workmanship that made the theoretically expected results of the test possible.

On the basis of the test results discussed in this chapter, it may be concluded that the experimental collapse

pressure of 2300 psi (obtained from the collapse of shell assembly shown in Fig. 13) represents the collapse pressure by general instability of shells with physical dimensions and material properties specified in Fig. 9.

CHAPTER IV

THEORETICAL CONSIDERATIONS AND DISCUSSION

4.1 STRUCTURAL ANALYSIS OF THE SANDWICH SHELL

The derivation of an equation describing the safe load of a novel structure can generally be approached from two diametrically opposite viewpoints. One viewpoint is based on the position that an equation describing the safe load for any new structure can be derived from basic tenets of statics and theory of elasticity if a thorough analysis of the distribution of loads and boundary conditions has been previously made. The other viewpoint rests on the supposition that any new structure can be considered to be a combination of several structural elements for whose load carrying capabilities solutions already have been obtained.

If approaches to the solution of problems could be characterized by one word, then the first viewpoint should be called scientific and the second, an engineering viewpoint. Both have their value depending on the aims of the investigation. The scientific type of theoretical formula derivation has its place and value when the aim of the

whole investigation is the discovery of a basic general set of equations describing adequately all cases to which the derived equation may be logically applied. When the aim of the investigation, however, is only the discovery of a specific engineering structure, then the engineering approach is much more desirable, the emphasis being on the utilitarian value of the formula and not on its value as a contribution to the theoretical body of knowledge.

Since this investigation was initiated by the sponsor to acquire an engineering design formula for the prediction of a particular mode of failure of a special type of structure, it was decided at the very initiation of the study that only the engineering type of approach was feasible. This decision was augmented by other reasons, such as the very limited amount of money, short period of time to conduct the investigation, and the sponsor's request to conclude the investigation with a concrete answer to his design problem.

4.1.1 Application of Bresse Ring Buckling Equation

The sandwich shell, when analyzed from the structural viewpoint, can be thought of either as an assembly of typical wide flange "I" rings, or as an outer and inner tube joined

at regular intervals by circumferential annular stiffening rings. Although equations describing general instability collapse of smooth tubes and circumferential rings exist, the structural interaction between these shell components is such that the collapse pressure of the assembly, due to general instability, cannot be equal to the sum of the individual collapse resistances of the inner tube, outer tube and circumferential stiffeners. Thus, it is fruitless to pursue the structural analysis approach which pictures the shell as composed of inner and outer smooth tubes joined by annular stiffening rings. The only avenue of attack left that logically promises a solution to the investigation problem is the one in which the shell is considered to be made up of infinitely repeatable typical wide flange "I" sections (Fig. 27).

When the hypothesis is made that the shell is in reality only a series of wide flange "I" rings, then it also follows that the overall collapse resistance of the shell to external pressure is equal to the buckling resistance of the structural module, the wide flange "I" ring. Proceeding on this assumption, the overall collapse resistance of the shell can be determined if the buckling strength of a single

TYPICAL SECTION
OF
CIRCUMFERENTIALLY STIFFENED SANDWICH SHELL

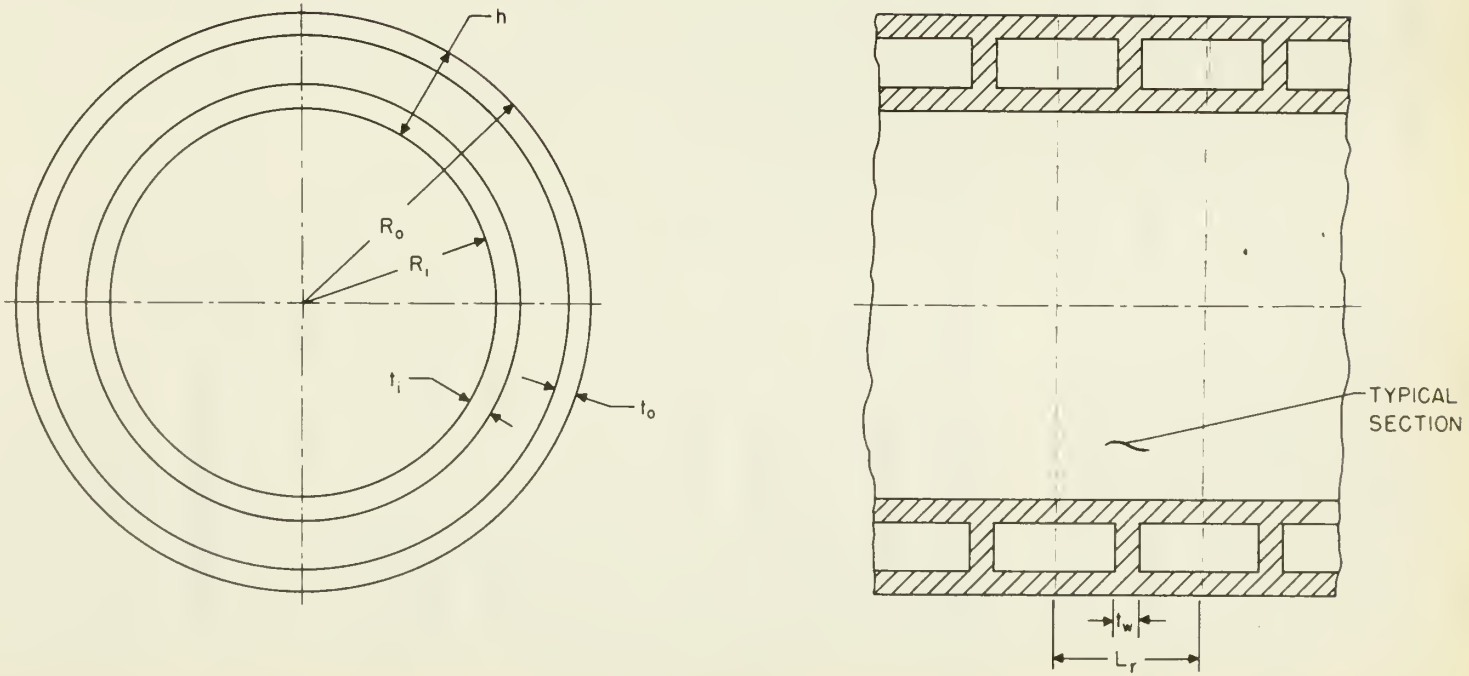


FIGURE 27

wide flange "I" ring is known. Fortunately, the problem of stability of a ring under uniform radially applied external loading was solved long ago by Bresse[95] and extended to plastic strain regions by Engesser[98]. The difference between the loading of Bresse's ring and that of a typical shell "I" ring here under investigation lies in the superimposition of axial load upon the ring along its outer and inner circumference.

The expression for the uniformly applied radial loading that produces radial buckling of the ring has been very lucidly presented by Timoshenko[114:186] and his notation is used here in the derivation of Eq. (8), the general instability equation, for the sandwich shells. The general instability equation which is derived here represents actually a semi-empirical adaptation of the Bresse[95] ring buckling theory, Eq. (6), to the buckling of sandwich shells by general instability. The adaptation is performed on the basis of structural similarity while the proof for the validity of the adaptation is given by experimental data.

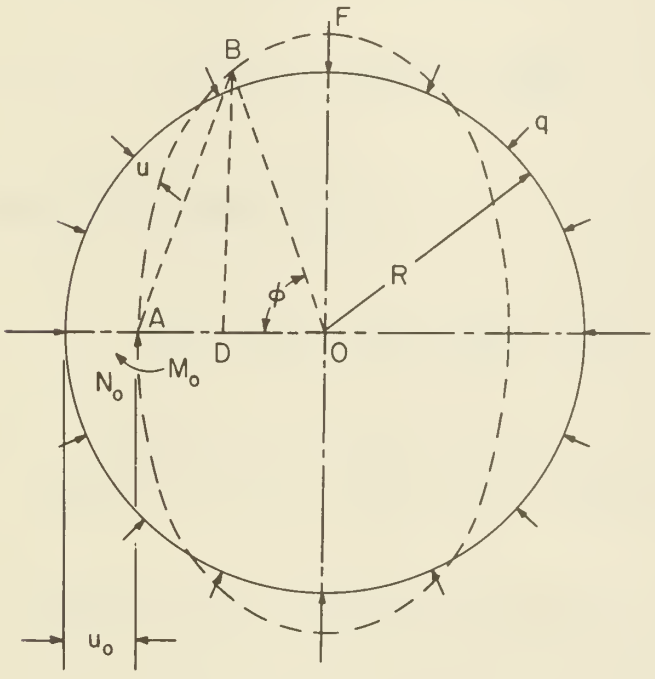
4.1.2 Buckling of Rings Under Radial External Pressure

The radial uniform pressure which causes a circular ring to become unstable and buckle is defined as the critical

BUCKLING OF CIRCULAR RING

UNDER UNIFORM RADIALLY APPLIED PRESSURE

(TIMOSHENKO, STRENGTH OF MATERIALS, PART II, 3RD EDITION, FIG. 120, P. 186.)



q = EXTERNAL PRESSURE, LBS/INCH OF WIDTH AND CIRCUMFERENCE
R = RADIUS OF CENTER LINE OF RING, INCHES
u = RADIAL DISPLACEMENT DURING BUCKLING
u_o = RADIAL DISPLACEMENT FOR CROSS SECTION A
M_o = BENDING MOMENT AT CROSS SECTION A
$N_o = q(R-u_o)$ = LONGITUDINAL COMPRESSIVE FORCE AT CROSS SECTION A

FIGURE 28

pressure. When such pressure is applied to the ring (Fig. 28) the ring can be assumed to buckle into an elliptical form as indicated by the super-imposed elliptical shape on the same figure.

The bending moment at any selected cross-section B of the buckled ring can be expressed as:

$$M = M_o + q\overline{AO} \cdot \overline{AD} - \frac{q}{2}\overline{AB}^2 \qquad (a)$$

Considering the triangle AOB,

$$\frac{1}{2}\overline{AB}^2 - \overline{AO} \cdot \overline{AD} = \frac{1}{2}\left(\overline{OB}^2 - \overline{AO}^2\right)$$

or

$$\frac{1}{2}\overline{AB}^2 - \overline{AO} \cdot \overline{AD} = \frac{1}{2}\left[(R-u)^2 - (R-u_o)^2\right]$$

As u is very small when compared to R, terms containing u^2 and u_o^2 can be neglected. Eliminating those terms,

$$\frac{1}{2}\overline{AB}^2 - \overline{AO} \cdot \overline{AD} = R(u_o-u)$$

Substituting this value into equation (a),

$$M = M_o - qR(u_o-u) \qquad (b)$$

To proceed further, a general equation has to be written that correlates the change in curvature with the magnitude of the bending moment. The differential equation for the deflection curve can be shown to be

$$\frac{d^2u}{ds^2} + \frac{u}{r^2} = - \frac{M}{EI} \qquad (c)$$

which becomes here

$$\frac{d^2u}{d\phi^2} + u = - \frac{R^2}{EI} \left[M_o - qR(u_o - u) \right]$$

or

$$\frac{d^2u}{d\phi^2} + u\left(1 + \frac{qR^3}{EI}\right) = \frac{-M_o R^2 + qR^3 u_o}{EI} \qquad (d)$$

The general solution of this equation is

$$u = C_1 \sin p\phi + C_2 \cos p\phi + \frac{-M_o R^2 + qR^3 u_o}{EI + qR^3} \qquad (e)$$

where the constants C_1 and C_2 can be determined from the conditions of the buckled ring at cross-sections A and F and

$$p^2 = 1 + \frac{qR^3}{EI} \qquad (f)$$

From symmetry, one can show that

$$\left(\frac{du}{d\Phi}\right)_{\Phi = 0} = 0$$
$$\left(\frac{du}{d\Phi}\right)_{\Phi = \frac{\pi}{2}} = 0 \qquad \text{(g)}$$

From the first of these conditions, one may obtain $C_1 = 0$ and from the second

$$\sin \frac{p\pi}{2} = o \qquad \text{(h)}$$

The smallest root of the Eq. (h) is

$$\frac{p\pi}{2} = \pi$$

or

$$p = 2$$

Substituting p = 2 into Eq. (f) the final expression for the uniform radial pressure causing a ring to buckle can be written as[114:189]

$$q_{ct} = \frac{3EI}{R^3} \qquad (6)$$

There are several other roots that will satisfy Eq. (h) but they all correspond to a larger number of waves in the buckled ring and thus give a higher magnitude q_{ct}. Equation (6) was first derived by Bresse and in this thesis is always referred to as Bresse ring buckling equation.

4.1.3 Buckling of Sandwich Shell Under Hydrostatic Pressure

Equation (6) is correct only for a single ring under radially applied external pressure. This means that there is no loading perpendicular to the plane of the ring and no external restraint for the buckling ring. When shells are considered which are made of typical wide flange "I" sections, then it becomes apparent that the flanges of an individual "I" ring are restrained from distortion by the adjacent "I" rings and are subjected not only to radial loading but also to axial loading. The restraint on "I" ring flanges and the superimposed axial loading must be taken care of in some manner as otherwise spurious answers will be calculated by the use of the above equation.

The simplest approach to the problem of restraint imposed on the flanges of an "I" ring by neighboring "I" rings is to assume that the cross-section of the flanges of

the "I" rings will not become distorted during bending since the neighboring adjoining flanges prevent it from distorting. This assumption is basically the same[96] that is made for the derivation of the buckling formula of a long smooth tube under uniform external radial pressure. Since the cross-section of the flanges will not be distorted during the bending of the rings under load, a new expression[114:189] must be substituted for modulus of elasticity in Eq. (6). Thus, the quantity

$$\frac{E}{1-\mu^2}$$

is substituted for E in Eq. (6) giving a new expression

$$p_{ct} = \frac{3EI}{R^3} \times \frac{1}{1-\mu^2} \times \frac{R}{R_o} \qquad (7)$$

The factor $\frac{R}{R_o}$ is used here to correct for the large difference in magnitude between the outside shell radius and the radius of the ring's neutral axis. For thin-walled smooth shells, such a correction is not necessary but for thick walled or sandwich shells whose $h/R_o > .1$ such a correction is mandatory for it amounts generally to about 10 per cent of the uncorrected value of p_{ct}. Even with the correction shown,

Eq. (7) is not exactly correct because the flanges of the "I" rings do not constitute the whole "I" ring but only a part of it. Thus, the flanges of the "I" ring are restrained from distortion while the web of the "I" ring is not restrained. To correct Eq. (7), one must treat the stiffness of the web and flanges separately because of the different type of loads acting on them.

Such detailed correction of Eq. (7) is not necessary, however, due to the fact that when the moments of inertia of the web and the flanges are compared, it is found that the web's contribution to the magnitude of the moment of inertia of the "I" ring is negligible. Since its contribution to the stiffness of the "I" ring is negligible, there is really no need for correcting Eq. (7) and the equation can be left as it is written for the case where the whole cross-section of the ring is restrained by the neighboring rings of the shell.

In the derivation of Eq. (7) it has been assumed that the material of the ring follows Hooke's law faithfully from zero stress to fracture. There are very few materials that behave in such a manner and so the equation must be modified to take into account materials that do not follow Hooke's law.

Engesser[98] and others[112] have developed expressions that consider the deviation of materials from Hooke's law while predicting the buckling of structural members.

To calculate the collapse pressure due to general buckling of a ring or cylinder fabricated from aluminum, the Engesser solution must be used. The Engesser solution for the buckling of structures fabricated from materials that do not have linear stress-strain properties consists of substituting the tangent modulus of elasticity for the modulus of elasticity in Eq. (7). There is little experimental data available on the correctness of Engesser solution as applied to the collapse of tubes or rings, but some data has been accumulated on its application to the buckling of slender rods (Fig. 29). One can see from the graph that the experimental points follow closely the theoretical curve predicting the buckling of the slender rods on the basis of the modified Euler's equation where the modulus of elasticity is replaced by the tangent modulus of elasticity. Realizing that both the buckling of rods and of shells is based on the same structural parameters, it is felt that the Engesser solution will hold equally well for tubes and composite shells.

Further refinement to Eq. (7) can be introduced by substituting μ_s for μ, where μ_s is Poisson's ratio of the

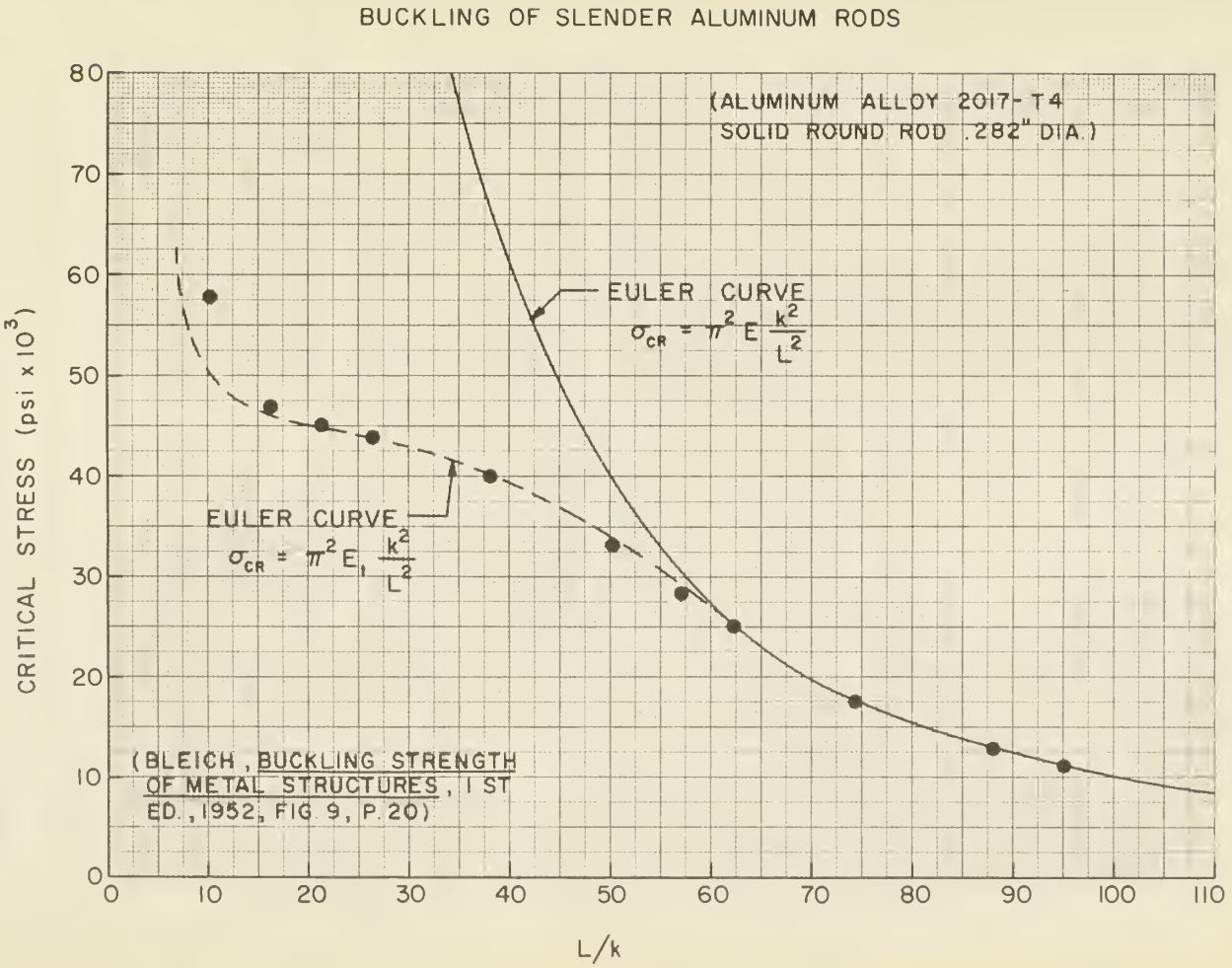

BUCKLING OF SLENDER ALUMINUM RODS
(ALUMINUM ALLOY 2017-T4
SOLID ROUND ROD .282" DIA.)
EULER CURVE
$\sigma_{CR} = \pi^2 E \frac{k^2}{L^2}$
EULER CURVE
$\sigma_{CR} = \pi^2 E_t \frac{k^2}{L^2}$
(BLEICH, BUCKLING STRENGTH OF METAL STRUCTURES, 1 ST ED., 1952, FIG 9, P. 20)
CRITICAL STRESS (psi x 10^3)
L/k
0
10
20
30
40
50
60
70
80
90
100
110

FIGURE 29

shell material at a given stress level. In the elastic strain region, Poisson's ratio changes very little with the increasing stress level, but in the plastic strain region the value of Poisson's ratio increases considerably over the value that it has in the elastic strain region[101]. When μ_s is used instead of μ in Eq. (7), the magnitude of the calculated collapse values for the plastic region may increase by as much as 18 per cent. However, the increase is generally about 5 per cent[101]. The difficulty in applying this correction lies in the scarceness of published data on the change of μ_s with the change in the stress level and thus, it is usually disregarded. The failure to use μ_s instead of μ places some of the calculated collapse pressure values in error, but since the error makes the calculated values smaller, it is accepted as a safe and conservative practice.

The modified Bresse equation for buckling of rings, Eq. (7), although specifically derived only for radial loading of the rings, can be used, also, to predict the general instability of sandwich shells under the joint action of axial and radial external pressures. The applicability of Eq. (7) to either uniform radial pressure or combined uniform axial and radial pressures (hydrostatic pressure) is based on the

fact[93] that it requires a much greater axial pressure acting alone than a radial pressure or combined axial and radial pressure to induce buckling in a smooth cylinder. Since in a hydrostatic loading of a shell the axial and radial external pressures are of equal magnitude, the cylinder will become unstable due to radial pressure long before buckling will occur due to axial pressure on the cylinder. Although this has been proved experimentally and theoretically only for smooth cylinders, it is logical to expect that it will also apply to sandwich shells due to similarity of the relevant shell parameters.

The final version of Bresse's ring buckling equation, modified to include E_t, μ_s, and $\frac{R}{R_o}$, can be written now as

$$p_{ct} = \left(\frac{3I_{nx}}{R^3}\right) x \left(\frac{1}{1-\mu_s^2}\right) x \left(\frac{R}{R_o}\right) x E_t \qquad (8)$$

and will be used in this thesis to calculate the general instability collapse of sandwich shells.

When the shell material follows Hooke's law up to a definite yield point and then becomes plastic without strain hardening (like mild steel), Engesser's solution does not apply and instead Southwell's[112] or some other modification

must be applied. Since this investigation does not concern itself with shells fabricated from such materials, the necessary modifications to Bresse equation will not be discussed here. Southwell[112] Ilyushin[102] and Von Karman[103] studied this problem.

4.2 COMPARISON OF EXPERIMENTAL AND THEORETICAL RESULTS

Both shells assembled into one vessel (Fig. 13) collapsed simultaneously under the application of external hydrostatic pressure. Since both of the shells collapsed simultaneously by general instability, the collapse pressure may be considered as an average and representative for Model A & A' shells and not a singular value that may differ from the average.

Figure 30 compares the collapse pressure calculated on the basis of Eq. (8) with experimentally obtained general instability pressure of aluminum sandwich shells. Figure 30 shows that the theoretical collapse pressure values for the two shells, after correction for end effects, almost coincides with the experimental value. The graph actually shows the relationship between the overall depth of the sandwich wall and the shell's collapse pressure, if the cross sectional area

of the sandwich wall remains constant while the depth of the wall varies. Such a graph is especially useful during the design of sandwich shells and Fig. 30 has been used for just such a purpose during the design of Models A and A'. Having plotted Eq. (8), it is easy to select the optimum wall depth for a shell of given outside diameter, material and weight. The optimum sandwich shell wall depth (denoted in calculations by h, see Fig. 27) is represented here by the sandwich wall that provides the most rigidity for the shell while taking up the least of internal shell volume. The optimum wall depth for the aluminum shells is selected with the aid of Fig. 30, and for the acrylic resin shell, with the aid of Fig. 32 .

For the aluminum shells one must select such a point on the graph (Fig. 30) where the rate of gain in resistance to collapse is the least while the rate of increase of the wall depth h is the largest. Looking at the graph, one sees that such a point is located on the graph immediately after the change-over from the inclined linear slope of the graph to the almost horizontal slope of the graph. At this point, the rate of increase in resistance to buckling is very slow, while the rate of increase in the required wall depth is very

COLLAPSE PRESSURE OF AN INFINITELY LONG
CIRCUMFERENTIALLY STIFFENED SANDWICH SHELL
UNDER UNIFORM EXTERNAL PRESSURE

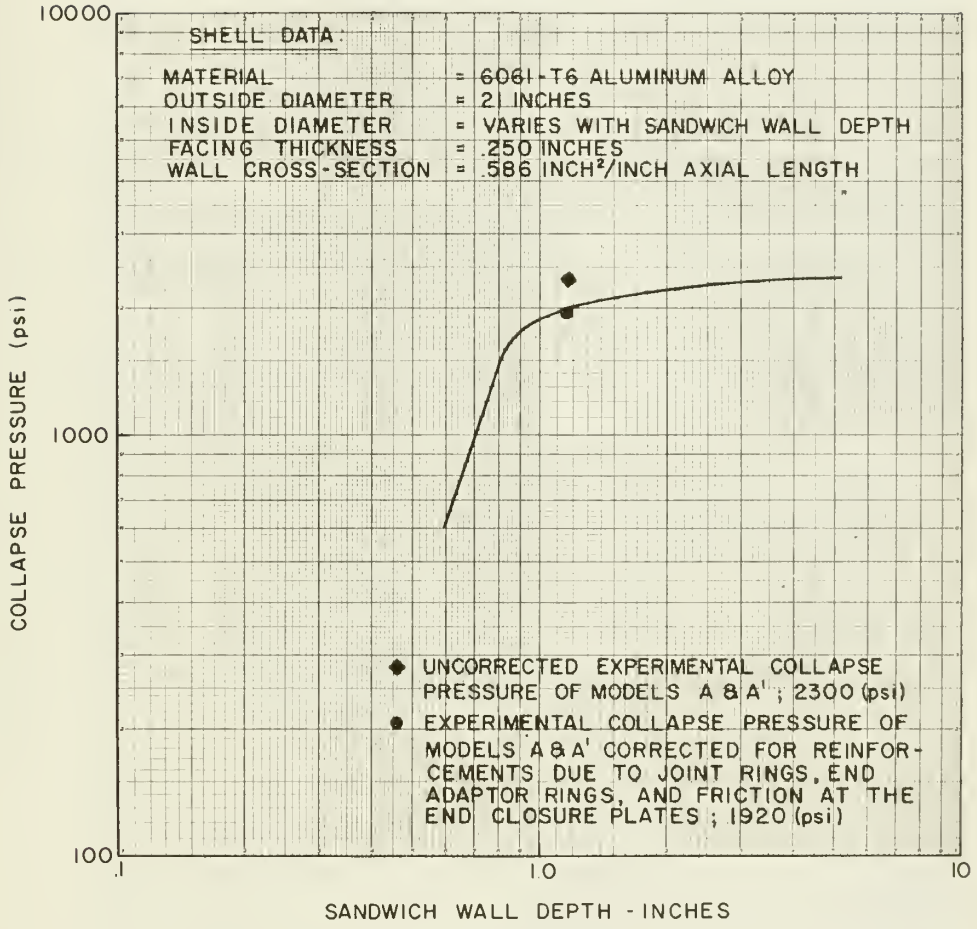


FIGURE 30

TANGENT MODULUS VS. COMPRESSIVE STRESS
6061-T6 ALUMINUM ALLOY

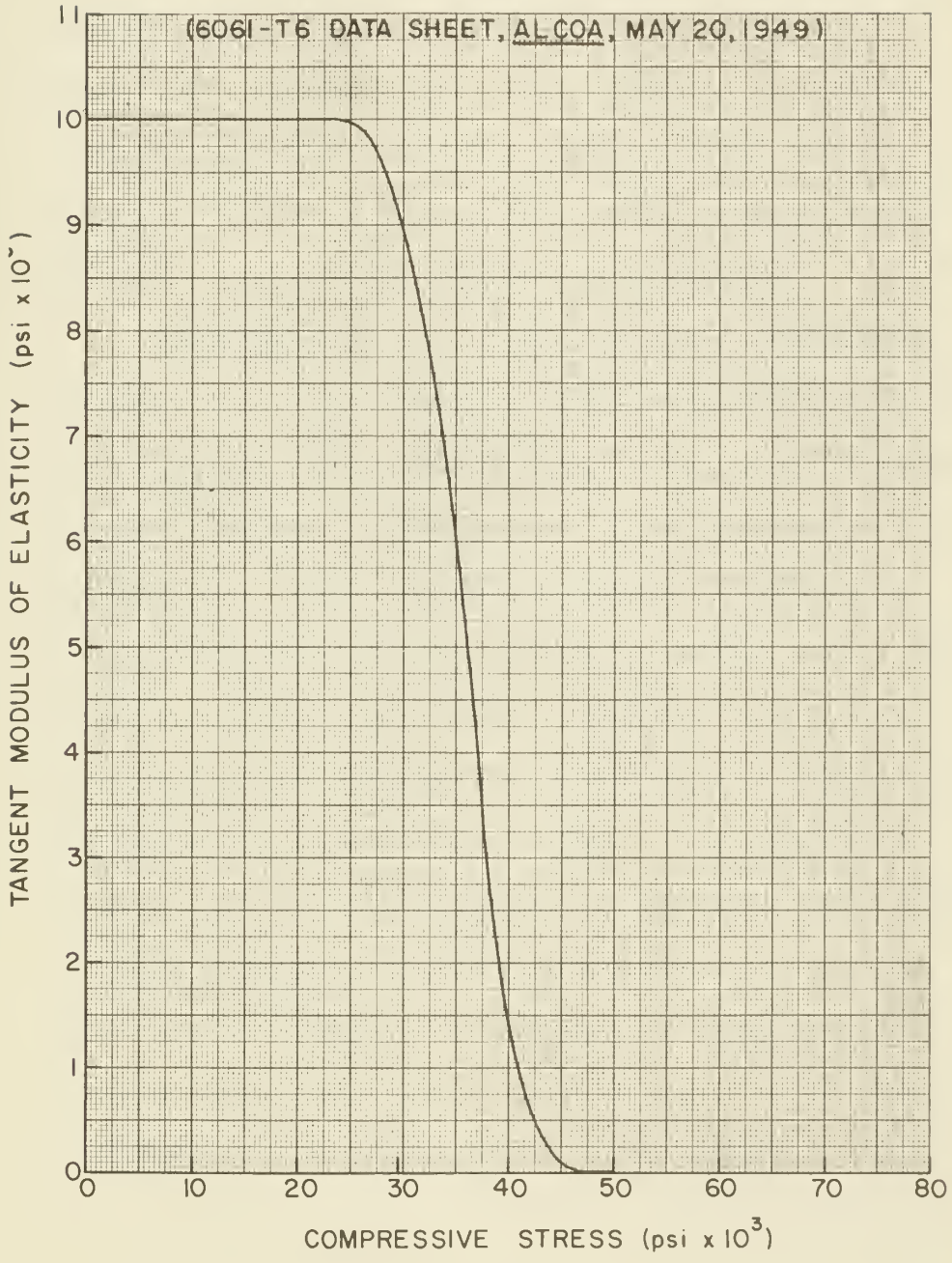


FIGURE 31

fast. This point on the graph represents the shell wall of such overall depth that the shell possesses also a maximized internal shell diameter. Selection of any other point on the graph will result in a shell that either has considerably lower collapse pressure with a slightly larger internal diameter, or a shell with slightly larger collapse pressure but with a considerably smaller inside diameter.

To prepare the plot of Eq. (8) it was necessary to obtain data on the behavior of both the 6061-T6 aluminum alloy and the acrylic resin from which the shells were fabricated. The data required for the plotting of Eq. (8) consist of a stress-strain curve, a tangent modulus of elasticity vs. stress curve, and a Poisson's ratio vs. stress curve. All of these above mentioned curves except the last one, have been obtained for aluminum from Alcoa Research Laboratories and the most important one is reproduced here in Fig. 31. Since a literature search failed to unearth any data on the change of Poisson's ratio with stress for this alloy and funds were not available to determine it experimentally, the curve of Fig. 30 was plotted using Poisson's ratio obtained from Alcoa for zero stress level on the assumption that the error introduced by this simplifying assumption is only of

minor magnitude. The assumption that the error introduced by the use of μ instead of μ_s is small is based on the known change of the Poisson's ratio for the 2014-T6 aluminum alloy[101]. The μ_s of this alloy increases to .4 in the intermediate plastic strain region. If the μ_s of 6061-T6 aluminum alloy also became .4 in the intermediate plastic strain region, then failure to take this into account will introduce only a 6 per cent error in the calculated collapse pressure of the shell. The data for the determination of tangent modulus of elasticity for acrylic resin was obtained experimentally.

When comparison is made between the theoretical and experimental collapse pressure values one must differentiate between actually obtained experimental values and the corrected experimental values. The actually recorded experimental collapse pressure values must be corrected to take into account the reinforcement of the shell by the presence of individual shell joints, assembly end adaptor rings, and friction end closure plates. If correction was not made for this strengthening effect, the experimentally obtained collapse values would not represent the collapse value of a long shell but of a short section stiffened at the ends, for which Eq. (8) is not applicable.

The correction of the raw experimental collapse data was performed according to Eqs. (2-6) as derived in this thesis. The correction calculations for the aluminum shells are shown below.

The experimental collapse pressure of Models A and A' = 2300 psi. Taking Eq. (3) the increase in the rigidity of the shell assembly due to friction end closure plates is

$$\Delta p_{ce} = \frac{.6R_o^2 p_{ce}}{RL} = \frac{(.6)(10.5^2)(2300 \text{ psi})}{(9,912)(70.500)} = 217 \text{ psi}$$

The increase in the rigidity of the shell assembly due to the shell assembly end adaptor rings at the closure plates is Eq. (4)

$$\Delta p_{ce} = \frac{2(p_{ctr}-p_{cts})}{L} = \frac{2(3910-2000)}{70.500} = 54 \text{ psi}$$

where the collapse pressure of the end adaptor ring is

$$p_{ctr} = \frac{3E_t I_{nx}}{(R)^2 R_o} = \frac{3 \times 10^7 \times .135}{(9.912)^2(10.5)} = 3910 \text{ psi}$$

and the collapse pressure of the typical "I" ring shell wall is

$$p_{cts} = \frac{3E_t I_{nx}}{(1-\mu_s^2)(R)^2 R_o} = \frac{3 \times E_t \times .111}{(1-.3^2)(9,912)^2(10.5)} = 2000 \text{ psi}$$

The increase in the rigidity of the shell assembly due to individual shell joints is Eq. (5)

$$\Delta p_{ce} = \frac{4(p_{ctj}-p_{cts})}{L} = \frac{4(3910-2000)}{70.500} = \underline{108 \text{ psi}}$$

where the collapse pressure of individual shell joints is also

$$p_{ctj} = \frac{3E_t I_{nx}}{(R^2)(R_o)} = \frac{3 \times 10^7 \times .135}{(9.912)^2(10.5)} = \underline{3910 \text{ psi}}$$

The collapse pressure corrected for end conditions and joint reinforcement can be obtained as follows:

Actual Experimental Collapse Pressure	= 2300 psi
Friction Plate Reinforcement	= -217 psi
End Adaptor Rings Reinforcement	= - 54 psi
Individual Joint Reinforcement	= -108 psi
Corrected Collapse Pressure	= 1921 psi

The difference between the corrected experimentally obtained collapse pressure and the collapse pressure predicted on the basis of Eq. (8) is 80 psi, less than 5 per cent and thus, satisfactory for engineering design purposes.

The corrected collapse pressure value of Fig. 30 shows very close agreement with the collapse value theoretically

calculated on the basis of modified Bresse Eq. (8). Realizing that the assumptions on which some of the corrections are based may contain some inaccuracies, one cannot place absolute confidence in the corrected collapse pressure that almost coincides with the pressure predicted by the formula. Coincidence of the two values is not construed here as absolute proof that Eq. (8) predicts collapse pressure of sandwich shell by general instability.

Further evidence that Eq. (8) predicts the general instability collapse of sandwich shells is found in the experimental work performed by the David Taylor Model Basin and also by calculating the collapse pressure of the already described acrylic resin sandwich shell that was imploded in the introductory test to this investigetion (Fig. 32). The experiments performed at the David Taylor Model Basin utilized steel shells of similar sandwich construction but different h/D ratios, and their collapse pressures, when calculated with the help of Eq. (8), also agree with the experimental collapse pressures.

Taking all this experimental evidence into consideration, it can be stated that there exists a sufficient amount of experimental evidence to substantiate the hypothesis that

COLLAPSE PRESSURE OF AN INFINITELY LONG
CIRCUMFERENTIALLY STIFFENED SANDWICH SHELL
UNDER UNIFORM EXTERNAL PRESSURE

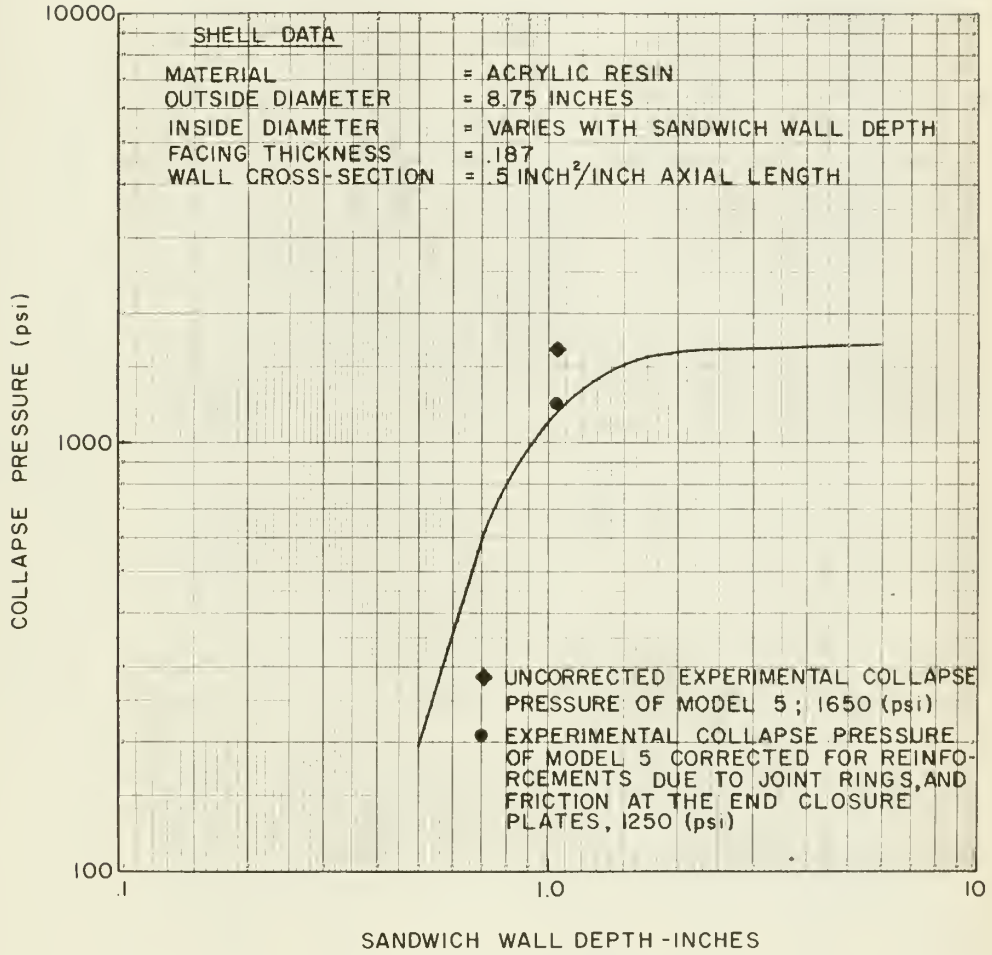


FIGURE 32

COMPRESSIVE STRESS - STRAIN CURVE
FOR
ACRYLIC RESIN USED IN CONSTRUCTION OF SHELL COMPARISON SERIES

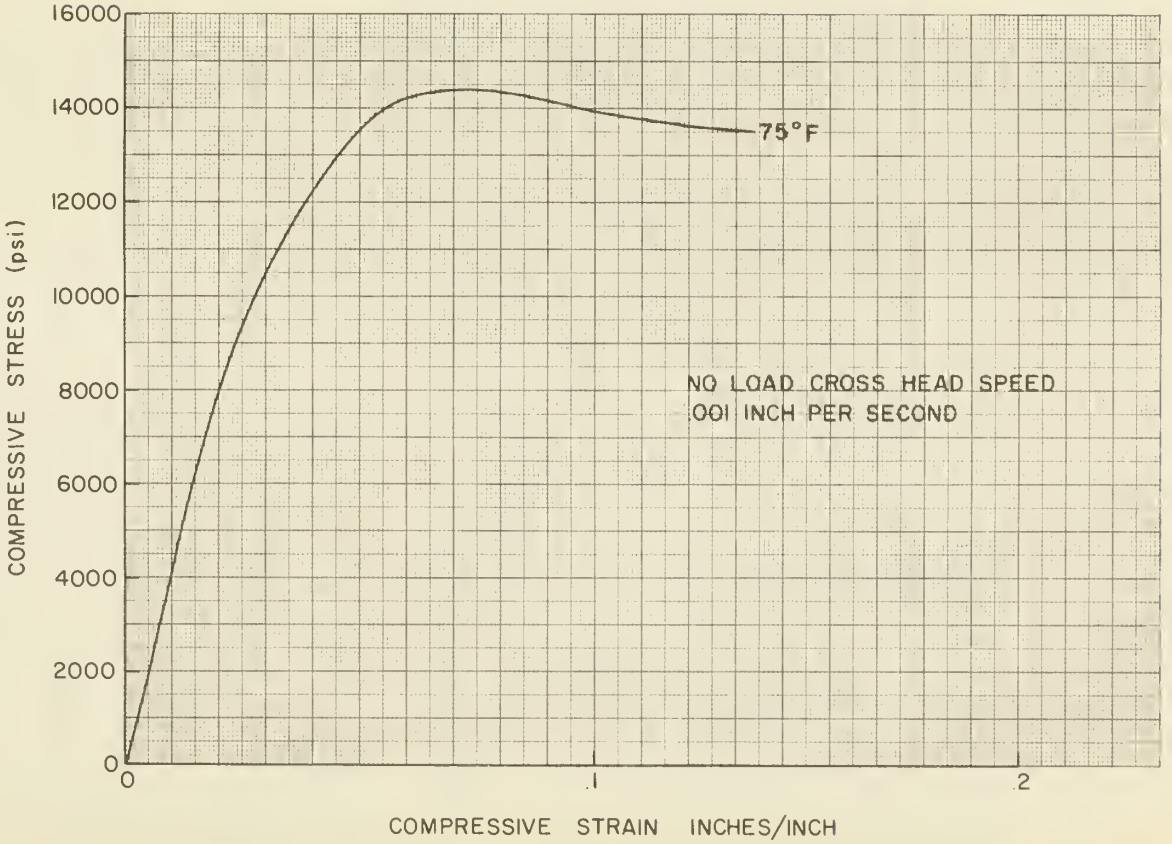


FIGURE 33

Eq. (8) accurately predicts for engineering design purposes the general instability collapse of sandwich shells.

4.3 SHELL COMPARISON INDICES

4.3.1 Basis of Comparison

When comparing different shell designs or identical shell designs but fabricated from different materials, it is very useful to have some kind of a yardstick that would make the comparison of one shell versus another more meaningful and objective. Many approaches have been tried, including the weight per unit length and the weight per unit length at a given displacement. Today only a few are accepted for it is very difficult to define an index that would hold for different shell diameters, length between bulkheads and construction materials.

The difficulty in defining a good structural index for the shell lies primarily in the multitude of parameters that could be logically selected as shell performance indices. Some of the parameters that could be used for comparison of shells subjected to external pressure are: weight, displacement, internal usable volume, collapse pressure and the yielding of the material. The basic requirement of a good

parameter is that it must apply, first of all, to shells of different diameters, different types of end support and for different construction materials without favoring any one of these variables.

At the start of this discussion it is important to state one basic truth that is too often overlooked when comparing shells with each other. Comparisons, to have any meaning at all, must be performed between shells of infinite length as otherwise end conditions of support unduly influence the collapse pressures. Too often one finds in the literature weight comparisons of shells that do not have identical bulkhead spacing. Even a cursory study of Kendrick's[106] general instability equation for smooth shells reinforced with circular rings will prove beyond doubt that neglecting the parameter of bulkhead spacing will introduce such a serious error as to make the comparison completely worthless. Since there does not exist a really accurate way of taking the bulkhead spacing into consideration when shells of novel design are compared to shells of proven design it is only prudent to set up the experimental comparison in such a way that the spacing of bulkheads becomes immaterial.

The bulkhead spacing is of no importance if all the shells are compared to each other on the basis of an infinitely long shell, that is a shell of such length that the method

of end closure used during the pressure testing does not exert any influence on the experimentally obtained collapse pressure. The distance between bulkheads that is equivalent to an infinitely long shell varies with the h/D ratio, but a spacing with $L \geq 5D$ will satisfy most practical shell h/D ratios.

If budgetary reasons, as is often the case, do not allow experimental comparisons of shells longer than five diameters, then the weight of the bulkheads has to be taken into consideration as most shells of length less than 5D derive their major share of rigidity from the bulkhead stiffness. If the foregoing practice is followed in selecting the lightest shell a good foundation is provided for all kinds of structural efficiency indices.

Keeping this fact in mind, several methods for scaling the weight of shells are discussed and their good and bad points noted. All of the diameter, weight and pressure scaling of shells that are developed here are for shells of infinite length as otherwise the scaling operation becomes very complicated.

4.3.2. Scaling of Shell Weight

It is very easy to scale the weight of a shell if the collapse pressure remains constant while the shell diameter

is varied. When only the stability of a smooth long shell is considered, the increase of wall thickness is directly proportional to the diameter increase, as can be seen from Eq. (7), from which after some algebraic simplifications, one obtains

$$p_{ct} = \left(\frac{2E}{1-\mu^2}\right)\left(\frac{t}{D}\right)^3 \qquad (a)$$

When the thickness is scaled from one diameter to another by the foregoing equation

$$p_{cta} = K\left(\frac{t_a}{D_a}\right)^3 \quad \text{and} \quad p_{ctb} = K\left(\frac{t_b}{D_b}\right)^3 \qquad (b)$$

$$\text{where } K = \left(\frac{2E}{1-\mu}\right) \quad \text{and} \quad p_{ctb} = p_{cta}$$

Using Eq. (b)

$$\frac{p_{cta}}{p_{cta}} = \left(\frac{K}{K}\right)\left[\frac{\left(\frac{t_a}{D_a}\right)^3}{\left(\frac{t_b}{D_b}\right)^3}\right] \qquad (c)$$

Simplifying Eq. (c)

$$1 = \left[\frac{\left(\frac{t_a}{D_a}\right)^3}{\left(\frac{t_b}{D_b}\right)^3}\right] \quad \text{or} \quad t_b = t_a\left(\frac{D_b}{D_a}\right) \qquad (d)$$

However, since the weight does not only depend on the wall thickness but also on the shell diameter, it will not increase linearly but as the square of the diameter ratio. Then if $p_{cta} = p_{ctb}$

$$W_b = W_a \left(\frac{D_b}{D_a} \right)^2 \tag{9}$$

This proportional ratio, although derived on the basis of general instability of smooth shells, holds equally well if the yielding of the material of the smooth shell is taken as the basis of the proportionality. This fact is of great importance for on that basis the weight comparison of two long smooth shells of different diameters and materials (but of equal collapse pressures) will be the same regardless of whether the instability or the strength of the material is taken as the basis of the scaling operation.

To compare the weight of two long smooth shells fabricated from different materials and having different collapse pressures the scaling Eq. (9) is not sufficient and the differential in pressure of the two shells has to be considered by using an additional increase or decrease of shell thickness. If the stability of smooth shells is taken as the basis of the criterion, then Eq. (9) must be written

$$W_b = W_a \left(\frac{D_b}{D_a}\right)^2 \left(\frac{p_{ctb}}{p_{cta}}\right)^{1/3} \tag{10}$$

If on the other hand stability is disregarded, and only the compressive strength is considered, then

$$W_b = W_a \left(\frac{D_b}{D_a}\right)^2 \left(\frac{p_{ctb}}{p_{cta}}\right) \tag{11}$$

Depending on the mechanical properties of the shell material, one or the other expression may be selected for a given external pressure range. If the calculated pressure to which a shell is subjected is varied from the elastic instability region of the shell to the failure region of the material, then the predicted weight of the shell has to be computed by both Eqs. (10) and (11), where Eq. (10) is used for defining elastic instability and Eq. (11) for defining of material failure. Since the collapse pressure of one of the two shells which are being compared may lie in the elastic instability range while the other lies in the material yield range, the comparison based on Eqs. (10) or (11) will be unfair to one of the shells. Which one of the shells will be penalized in the weight comparison depends, of course, on which of the two formulas is used for the scaling of the weights.

Even if Eqs. (10) and (11) are applied properly the comparison is good only for indicating what the approximate weight of a shell structure of given diameter and collapse pressure will be when it is scaled from a shell of different dimensions and pressure capability. The comparison bases of Eqs. (10) and (11), however, is not valid for the quantitative evaluation of the efficiency in the bracing of shell structure against implosion. In other words, when in a comparison one shell is lighter than another shell, after proper scaling operations have been performed by the use of Eqs. (10) and (11), then this may be due to one of the following factors:

(1.) Better design practice

(2.) Use of material with better mechanical properties

(3.) Combination of (1) and (2)

As long as only the weight of the shell of given dimensions and pressure capability is considered as the comparison criterion, the judicious use of both the above mentioned scaling equations will help to make a valid comparison; it will help to select the lightest shell. Although the scaling equations were derived specifically for long tubes and long smooth shells, they also hold approximately well for long

shells of rib reinforced or sandwich construction, Eq. (11) giving values that are in best agreement with experimental scaling data.

Some workers investigating the strength of shells were dissatisfied with the methods of comparison used and suggested the use of structural efficiency factors as indices of shell design desirability. One of the better known structural efficiency factors was proposed by Wenk[120] and is defined by

$$E_s = p_{ce}\left(\frac{V_i}{W_i}\right) \qquad (12)$$

where p_{ce} = experimental collapse pressure

V_i = displacement per unit length

W_i = weight per unit length

Although it is often referred to, particularly in the U. S. Government publications, it does not satisfy all the requirements that have been discussed above. Its chief drawback lies in the fact that when two shells with different collapse pressures, diameters, and construction materials are compared then a higher rating obtained for one of the shells may be due either to one, or all of these factors:

(a.) Better material

(b.) Shorter bulkhead spacing

(c.) More efficient method of placing the stiffeners inside the shell

Considering all the scaling difficulties, and realizing that in all of the formulas discussed the use of superior construction material may give an excellent pressure weight to displacement ratio, it can be postulated that structural design indices of real value for shells are those that are not influenced by the effects of scaling and the mechanical and physical properties of materials. Only when the dubious scaling of weight and the compromising inclusion of mechanical and physical material properties in the structural index are dropped does the structural index become an actual measure of the efficiency with which a given design utilizes material to secure sufficient rigidity for the shell to withstand uniform external pressure.

It can be stated that at present there exists no comparison method that takes into account all the possible shell design parameters to give sound rating for the different methods by which a shell is stiffened against failure when subjected to external pressure. Many carefully planned and executed shell design experiments will be necessary before a valid method for the comparison of different shell design theories will be found that will permit the rating of shell designs.

CHAPTER V

SUMMARY AND CONCLUSIONS

5.1 STATEMENT OF THE PROBLEM

Circumferentially stiffened cylindrical sandwich shells subjected to uniform external pressure often fail by the collapse mechanism of general instability. There exists a need for engineering design formula that would predict with adequate accuracy the critical pressure at which general instability in such shells occurs.

5.2 ORIGIN AND IMPORTANCE OF THE PROBLEM

Modern engineering requires for many applications cylindrical shells with very high external pressure to weight ratios. The well-known and well-developed ring reinforced shells have reached their limit of improvement and their pressure to weight ratio cannot be improved any further due to limitations in the methods of reinforcement by which they derive their rigidity.

There is a consensus among shell designers and some experimental supporting data backing up this opinion, that sandwich shells will yield a much higher pressure to weight

ratio than it is feasible with ring reinforced smooth shells. The increased rigidity of sandwich type shell walls makes possible the use of very strong materials without the occurence of local buckling. Among the many conceivable types of sandwich shells, the circumferentially stiffened one has been selected by the sponsor of this investigation as the most desirable since it fulfills all of his requirements for a shell subjected to external pressure.

Upon a thorough search of sandwich shell literature it was concluded that no experimentally proven design equations exist for the design of circumferentially stiffened sandwich shells. The purpose of this study was to obtain a theory with experimental support that provides a simple relationship for accurately predicting the general instability collapse of circumferentially stiffened sandwich shells.

The investigation is important for two reasons:

(1.) Intensive literature search has failed to show other investigators who have verified experimentally their proposed theories for the general instability of circumferentially stiffened sandwich shells.

(2.) Among all the proposed equations, the equation developed herein is shown to be the simplest and most useful for engineering purposes.

5.3 PROCEDURE OF THE INVESTIGATION

An engineering type approach was used in the development of a theoretical general instability equation. Instead of starting with the fundamentals of theory of elasticity and elastic stability of a sandwich shell structure, a similarity concept between the investigated structure under external pressure and a wide flange "I" ring under radial pressure was formulated and used as a foundation for further work.

On the basis of the modified Bresse[95] Eq. (8) for the buckling of rings, postulated in this thesis as applicable to circumferentially stiffened shells, one acrylic resin (8.75" diameter x 15.7" long) and two aluminum (21" diameter x 33.5" long) shells were designed and fabricated.

The acrylic resin sandwich shell was tested to destruction without any instrumentation, the objective of the test being a preliminary confirmation of Eq. (8) used as the basis of design for this shell. The two aluminum sandwich shells, designed on the basis of the same Eq. (8), were, prior to testing, coupled together to form one long shell assembly. After instrumenting with electric resistance strain gages, the shell assembly was provided with friction type end closures and also submitted to implosion testing.

5.4 RESULTS

Both of the circumferentially stiffened aluminum (21" diameter and 33.5" long) sandwich shells failed by the mechanism of general instability at 2300 psi of external pressure. When the experimentally obtained collapse pressure of 2300 psi is corrected for the reinforcing effect of joint rings and friction type end supports the correct collapse pressure for infinitely long shells of identical construction is found to be 1920 psi. A comparison of the corrected experimental collapse pressure of 1920 psi with the collapse pressure of 2000 psi calculated on the basis of modified Bresse equation for buckling of rings shows little difference between these values. Similar results are obtained when the corrected experimental collapse pressure of the acrylic resin circumferentially stiffened sandwich shell is compared to the collapse pressure calculated on the basis of Eq. (8). The difference between these values is less than 5 per cent (Figs. 30 and 32).

5.5 CONCLUSIONS

On the basis of structural similarity between long circumferentially stiffened sandwich shells and a single wide flange "I" ring, the Bresse[95] general instability

equation for rings has been modified by the writer to encompass and predict the buckling of circumferentially stiffened shells to uniform external pressure.

The actual implosion pressure of two 21" diameter 33.5" long circumferentially stiffened aluminum and one 8.75" diameter 15.7" long acrylic resin sandwich shells support conclusively the modified Bresse general instability equation for these shells. The differences between calculated and experimental critical pressures were found to be less than 5 per cent.

The theoretically derived and experimentally checked modified Bresse equation predicting the collapse pressure of circumferentially stiffened sandwich shells is expressed by Eq. (8) shown below.

$$p_{ct} = \left(\frac{E_t}{(1-\mu_s^2)}\right)\left(\frac{I_{nx}}{R^2 R_o}\right)$$

In the present form the above equation applies only to infinitely long shells fabricated from materials that do not possess a pronounced yield point. Infinitely long shells are defined in this thesis as shells whose collapse pressure is not dependent on their bulkhead spacing. It may be used also

with short shells if proper corrections for the influence of end conditions are made.

5.6 SUGGESTIONS FOR FURTHER RESEARCH

In this thesis a modified ring buckling equation was developed that accurately describes the general instability of long circumferentially stiffened sandwich shells. The knowledge of this equation should aid engineers in the calculation of collapse pressure for this type of sandwich shells.

There still remain many other modes of collapse by which these shells may fail when subjected to external pressure. The shells may fail due to local buckling of ring flanges, local buckling of ring webs, or local yielding of the shell material. These types of local failure represent areas of scientific interest for future investigations. Both theoretical and experimental approaches are necessary to arrive at proven workable design equations that will permit the engineer to design circumferentially stiffened sandwich shells with such physical and structural parameters that no type of failure will occur in the operating pressure ranges of the shell.

The biggest need at the present time is for a large body of well documented implosion test data for circumferentially stiffened sandwich shells of different lengths and diameters.

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